SPECULATIVE NOISE TRADING AND MANIPULATION IN THE FOREIGN EXCHANGE MARKET

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Abstract

We investigate the possibility that in the foreign exchange market uninformed speculators find it convenient to trade on noise in order to gain an informational advantage they can exploit in future. In a two-period model, we analyze the trade-off between the cost of the “informational investment” and the profits this brings about, studying the optimal manipulation strategy under different hypotheses on the activity of market participants. Our results give a possible explanation for the presence of noise trading in the foreign exchange market.

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1 Introduction

A large component of transactions in securities markets derives from noise traders. According to Black (1986), noise traders are agents who sell and buy assets on the basis of irrelevant information. These speculators do not possess inside or fundamental information and trade irrationally on noise as though this gave them an edge. Despite its irrational nature, noise trading represents an important aspect of the functioning of securities markets, because it reduces the risk of market crashes and facilitates transactions among agents. Indeed, if all traders were rational it would not be convenient to gather information, because prices would be fully revealing. Conversely, when noise traders are present, rational speculators will gain profits at the expense of the irrational ones. As in this case prices will not be fully revealing, there would still be an incentive to gather information, so that in practice noise trading may be beneficial for the efficiency of securities markets.

Different types of behavior are associated with noise and liquidity trading. These comprise hedging strategies, such as portfolio insurance and stop-loss orders, popular models of forecasting and trading, such as chart and technical analysis, and so forth. In particular, considering the market for foreign exchange, there is ample evidence of widespread use of chartism by traders. Indeed, on the basis of a survey conducted in the London foreign exchange market, Allen and Taylor (1990) point out that most traders consider chartism at least as relevant as fundamentalism in the formulation of exchange rate expectations in the short run. Likewise, Frankel and Froot (1987), using survey data in the foreign exchange market, find strong evidence of extrapolative expectations and attribute it to the utilisation of chartism by professional traders.

A problem with this description of noise trading in securities markets is its profitability. In fact, it is commonly believed that noise traders incur losses, because they buy when prices are high and sell when prices are low. Nevertheless, empirical studies question this thesis. Goodman (1979) compares the profitability of different forecasting techniques in the market for foreign exchange. In the seventies, the worst technically orientated forecasting technique was three times more profitable than the best fundamentally orientated one. Likewise, Schulmeister (1988) finds that large speculative profits of commercial banks in the market for foreign exchange are due to the utilization of technical trading rules. Levich and Thomas (1993) confirm the thesis that chartism is profitable employing a bootstrap approach, while Menkhoff and Schlumberger (1995) extend previous results to a longer period.

Therefore, explaining the use of chartism and other forms of noise trading represents an important

\footnote{Liquidity traders differ from noise traders in that their transactions are led by hedging needs and not by speculative reasons. Generally this distinction is lost in theoretical models of securities markets, while it represents an important element of our analysis.}
topic of research. Several explanations have been proposed. Frankel and Froot (1986) suggest that
if traders learn slowly the fundamental value of a currency, chartists can dominate fundamentalists
and lead the exchange rate on a bubble path. De Long, Shleifer, Summers and Waldmann (1990)
show that if irrational traders can bear more risk than other traders, they can gain larger profits
than risk-averse rational speculators. Froot, Scharfstein and Stein (1992) show that if traders have
short horizons, they may find it convenient to trade on the basis of information completely unrelated
to fundamentals. Palomino (1996) proves that in small markets in which investors are not price
takers, noise traders might hurt sophisticated traders more than themselves.

Our analysis suggests an alternative explanation, based on a particular mechanism of manipulation
of expectations and exchange rates. Even if this analysis could be applied to other dealer markets,
we refer principally to the foreign exchange market because it is there that most of the evidence on
the profitability of noise trading is concentrated. Moreover, it is in this market that the mechanism
of manipulation we suggest is most likely to take place.

Noise trading may be part of a valid strategy of manipulation and a source of speculative profits,
because it can bring about an informational advantage. This happens in a dealer market when
noise trading interferes with the learning process of dealers. In fact, assuming that dealers are
competitive and cannot distinguish between informed and uninformed trades, noisy market orders
placed by an uninformed speculator will reduce their ability to learn the fundamental information
contained in the order flow. On the other hand, the uninformed speculator will be able to extract
more of the fundamental information present in the flow of orders. Then, in subsequent periods
she can use her informational advantage to recoup the losses incurred with the initial noisy market
orders and also gain some profits.

One should point out that our contribution extends the analysis put forward by Madrigal (1996).
In fact, he considers a dealer market in which some speculator observes some of the market orders
placed by liquidity traders and acquires an informational advantage with respect to the dealers she
can exploit to gain speculative profits. Indeed, the main difference between the two contributions
is that whilst in his model the speculator possesses some non-fundamental private information, in
our analysis we discuss the case in which the speculator does not have any initial informational
advantage.

In what follows we consider a model of the foreign exchange market with private information on
the fundamental value of a foreign currency. This framework permits studying the impact of noise
trading on the learning processes of the market maker and the uninformed speculators and on the
stochastic process governing the exchange rate. In addition, we can analyze the trade-off between
the cost of noisy market orders and the informational advantage they provide and study under
which conditions they may be part of a profitable trading strategy of the uninformed speculators.
This paper is organized as follows. In Section 2 we show that an uninformed trader can gain an informational advantage placing random market orders in such a market. In the following Section we prove that under particular circumstances the speculator can gain positive profits. In Section 4 we extend our basic formulation in several directions to investigate the robustness of our results. A conclusion which summarizes the results of our analysis completes the paper, while an appendix gives detailed proofs of Propositions and Lemmas.

## 2 Noise in the Foreign Exchange Market

To analyze the effect of noise trading on the performance of the foreign exchange market, we consider a simple model based on the batch framework put forward by Kyle (1985). According to this framework in the market for foreign exchange a representative dealer (market maker) transacts a foreign currency with a population of customers before some news on the fundamental value of the currency is announced. A simple interpretation of this assumption is that this news concerns the money supply. For instance, in the United States decisions of intervention in the domestic monetary market are taken at the meetings of the Federal Open Market Committee. While actions immediately follow the meetings, communiqués on their conclusions are released one week later (Stein (1989)) and actual data on monetary aggregates are disclosed with a two-week lag (Tabellini (1987)). Assuming that monetary variables affect the fundamental value, uncertainty on the fundamental value can be a consequence of uncertainty on the current monetary policy.\(^2\)

We assume that trading takes place in a sequence of auctions or rounds of trading. Any round of trading is organized as follows. First, market participants place *anonymously* their market orders, which are batched and passed to the market maker. Then, since price competition enforces a zero-expected profit condition for the dealers, the market maker will set the exchange rate equal to the expected fundamental value given his information, according to a weak form efficiency condition.

We suppose that there are 2 rounds of trading before the fundamental value of the foreign currency, \(f\), is announced. In this way we can isolate a first period, in which a speculator invests in the acquisition of an informational advantage, from a second one, in which she uses it to gain speculative profits. A multi-period formulation of this model is not analytically tractable, but if manipulation is viable when only two rounds of trading are possible, then trading over several periods would

\(^2\)Lewis (1995), applying Granger Causality tests, finds that non-borrowed reserves influence the exchange rate, while Ito and Roley (1987) find a similar result for news on monetary variables.

\(^3\)In a simple monetary model the fundamental value of the exchange rate will be a function of the expected future values of the money supply. Assuming that the money supply follows an AR(1) process, the fundamental value is a linear function of the current money supply and therefore the uncertainty about the fundamental value corresponds to the uncertainty about the money aggregates.
facilitate it.

The fundamental value is given by a normally distributed random variable, $f$, where $f \sim N(s_0, \Sigma_0)$. We assume that the market maker cannot observe the fundamental value of the foreign currency, but that he receives in any round of trading $t$ ($t = 1, 2$) a total order flow, $x_t$, given by the following expression:

$$x_t = a_t(f - s_{t-1}) + \epsilon^t,$$

where $a_t > 0$ and $s_{t-1}$ is the exchange rate set in the previous round of trading.

This assumption implies that the order flow contains both noise and signal components. In fact, random market orders, $\epsilon^t$, placed by liquidity traders follow a white noise process of variance $\sigma^2_t$ and independent of the fundamental value. Simultaneously, informative market orders, $a_t(f - s_{t-1})$, are placed by an agent which possesses information on the fundamental value, $f$, and arbitrages against the undervaluation (overvaluation) of the foreign exchange, $f - s_{t-1}$. In practice, since it is very unlikely that individual investors possess a better knowledge of the fundamental value than professional dealers, this agent will be a central bank that has superior information on its current and future monetary policy. As suggested in the literature (Edison (1993)), central banks are often active in the market for foreign exchange with sterilised intervention operations aimed at signalling changes in the fundamentals of exchange rates.

In practice, we assume that when the central bank intervenes in the foreign exchange market it has to take the fundamental value as given. This implies that there exists a separation between foreign exchange intervention and other policy-making tools. In effect, both in Japan and in the United States foreign exchange and monetary policies fall under the jurisdiction of different institutions.4

The structure of the foreign exchange market does not correspond to that of the auction market considered by Kyle, but there are several reasons that can justify its use in the present context. First, this framework is elegant and powerful, as simple analytical solutions are easily derived and have intuitive interpretations. Moreover, the batch framework captures the most important aspect of the foreign exchange market: its lack of transparency. In fact, in both markets dealers cannot observe all market orders and prices cannot immediately incorporate all private information contained in individual trades. This “opaqueness” is fundamental for the functioning of the foreign exchange market and has two important implications for our analysis.

On one hand, a central bank can conceal its intervention operations, by placing market orders

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4The Minister of Finance in Japan and the Treasury in the United States have the authority over the foreign exchange policy, that is actually carried out by the respective central banks. Moreover, they also possess special foreign reserves funds (the Foreign Exchange Fund Special Account and the Exchange Stabilization Fund respectively) to implement their intervention policies.
through an elected commercial bank that trades on its behalf, and still convey signals to the market, as the transactions of this elected bank will be perceived by the rest of the market as noisy signals of the activity of the central bank. On the other hand, since any speculator can maintain her market orders secret, she can effectively manipulate market expectations and exchange rates without revealing her identity.

The weak form efficiency condition implies that the dealer will fix the exchange rate equal to the expected fundamental value of the foreign currency, given the history of the order flow. Thus, we have that \( s_1 = E[f|x_1] \) and \( s_2 = E[f|x_1, x_2] \). The application of the projection theorem for Normal random variables immediately gives a recursive expression for the exchange rate:

\[
s_t = s_{t-1} + \lambda_t x_t,
\]

with \( t = 1, 2 \). If one defines \( \Sigma_{t-1} \) as the conditional variance of the fundamental value in period \( t-1 \) given the history of the order flow, the liquidity coefficient, \( \lambda_t \), is:

\[
\lambda_t = \frac{a_t \Sigma_{t-1}}{a_t^2 \Sigma_{t-1} + \sigma^2}.
\]

Then, the conditional variance of the fundamental value in period \( t \) \( (t = 1, 2) \) is:

\[
\Sigma_t = (1 - a_t \lambda_t) \Sigma_{t-1}.
\]

We see from these equations that sterilised intervention does not need to be visible to alter exchange rates. In addition, we can say that even operations of limited scale can be effective. In fact, despite the massive volume of trading in the foreign exchange market, the activity of individual dealers remains within limited dimension. This means that with relatively small market orders a central bank can affect the quotes of a single market maker. Then, if this market maker has the reputation of receiving market orders from the central bank, inter-dealer transactions will propagate this effect to the rest of the market. In effect, in an empirical study Peiers (1997) confirms that certain commercial banks stand as market leaders during periods of central bank intervention.

Suppose, now, that an uninformed speculator enters in the foreign exchange market.\(^5\) She cannot observe the fundamental value, \( f \), but can trade anonymously and place unpredictable random market orders. This can happen if during periods of central bank activity, the speculator trades with the market leader, disturbing his learning process. In this respect, let us introduce the following Assumption.

\(^5\)Indeed, we could consider a group of \( M \) uninformed speculators. We leave the discussion of that case to Section 4 and consider now only the possibility that a single uninformed speculator enters in the market.
Assumption 1 In any round of trading $t$ the speculator can commit to use one from a set of noise trading technologies, $T$. Any element $t$ of $T$ will place at time $t$ a random market order $b_t \eta_t$, where \( \{b_t\}^2_{t=1} \) are non-negative constants and \( \{\eta_t\}^2_{t=1} \) is a white noise process of variance $\sigma^2_t$, independent of the fundamental value and of the liquidity trading process.

In practice, we assume that the speculator may access a series of automatic trading programmes, based on the processing of information unrelated to the fundamental value of the foreign currency. This is the case for investors using chart and technical analysis: their transactions are dictated by patterns observed in past exchange rates, which in efficient markets are not related to future changes in the fundamentals.\(^6\)

When a noisy trading technology $t$ is used the order flow becomes:

\[
x_t = a_t(f_t - s_{t-1}) + b_t \eta_t + \epsilon^t_t.
\]

Thus, in equation (2), the liquidity coefficient, $\lambda_t$, is now equal to:

\[
\lambda_t = \frac{a_t \Sigma_{t-1}}{a_t^2 \Sigma_{t-1} + \sigma^2_t}, \quad (4)
\]

where $\sigma^2_t = \sigma^2_t (1 + b^2_t)$. Because of the reduction in $\lambda_t$, which measures the informativeness of the order flow, the conditional variance of $f$ in equation (3) rises for any given value of $\Sigma_{t-1}$. This shows that the uninformed speculator can condition the learning process of the market maker: by injecting noise in the order flow she can determine the market maker uncertainty on the fundamental value.

Within the batch framework, customers cannot directly observe the order flow of the dealer. Anyway, the value of $x_t$ can be determined in equilibrium observing the exchange rate customers will pay or receive for one unit of the foreign currency in the round $t$ of trading (customers can simply invert equation (1) to obtain $x_t$). In other words, a speculator can “test the water” and obtain some useful information on the activity of the foreign exchange market by placing a market order with a dealer. On the contrary, posted quotes are generally only indicative and never represent firm values, so that they do not constitute an instrument of diffusion of information in the market.

Since the speculator will observe ex-post the realisation of her random market order, $x^s_t$, she can extract at the end of period $t$ the total market order of the other customers: $y_t = x_t - x^s_t$. In this way, she can form her own expectations of the fundamental value, $s^s_t$, which in parallel with equation (1) respects the following recursion:

\[
s^s_t = s^s_{t-1} + \lambda^s_t y_t, \quad (5)
\]

\(^6\)Notice that a mechanism of commitment is necessary to avoid problems of time consistency. This mechanism can be designed using some form of portfolio delegation as suggested by Biais and Germain (1997).
with \( s_0^s = s_0 \). If one defines \( \Sigma_{t-1}^s \) as the conditional variance of the fundamental value in period \( t - 1 \) given the information of the speculator, \( \lambda_t^s \) respects a formulation similar to equation (2):

\[
\lambda_t^s = \frac{a_t \Sigma_{t-1}^s}{a_t^2 \Sigma_{t-1}^s + \sigma_t^2}.
\] (6)

Obviously a recursion for the conditional variance similar to equation (3) holds:

\[
\Sigma_t^s = (1 - a_t \lambda_t^s) \Sigma_{t-1}^s, \quad \text{where} \quad \Sigma_0^s = \Sigma_0.
\] (7)

Then, studying equations (3) and (7) it is immediate to verify the following Proposition.

**Proposition 1** For any choice of \( \{a_t\}_{t=1}^2 \), if the speculator employs a noisy trading technology in period 1, \( b_1 > 0 \), then her degree of uncertainty on the fundamental value is smaller than that of the market maker in any round of trading: that is, for \( t = 1, 2, \Sigma_t^s < \Sigma_t \).

Proposition 1 implies that an uninformed speculator can exploit the uncertainty of the market maker on the information possessed by his clients to derive more precise information on the fundamental value of the foreign currency. She just needs to inject some noise in the order flow in the first round of trading and hence observe the exchange rates set by the market maker in the following one. This is sufficient to gain an informational advantage in the first period and to conserve it until the uncertainty on the fundamental value is resolved. The informational gain is obtained reducing the information contained in the order flow observed by the market maker, that is reducing its signal-to-noise ratio.

In substance, in the foreign exchange market a trader can contact a dealer, who stands as market leader during periods of central bank activity, and place a market order. This can achieve the joint objective of gathering some information on the most recent intervention operations and influencing the transaction price charged by the market maker.

To conclude this section, it is important to notice that the result outlined in Proposition 1 can be generalised in several directions. In fact, it would still hold if a group of \( M \) speculators entered in the market, if a multi-period formulation of our model were considered, if a more complex Guassian linear stochastic structure were imposed on the fundamental value and even if the central bank activity were endogenous. In any case, we discuss some of these extensions in Section 4.
3 Equilibrium with Manipulation

3.1 The Optimisation Problem of the Speculator

Let us assume the speculator is risk-neutral. Then, at any round of trading \( t, t = 1, 2 \), she maximises the expected value of her trading profits, \( E[\pi^s|I^s_t] \), where \( I^s_t \) is her information set at the beginning of period \( t \). We indicate with \( f \) the fundamental value of the foreign currency in period 2, with \( s_t \) the exchange rate fixed by the market maker and with \( x^s_t \) the market order of the speculator at time \( t \). As the fundamental value is publicly announced at the end of period 2, the speculator will realise the following total profits:

\[
\pi^s = \sum_{t=1}^{2} (f - s_t) x^s_t = \sum_{t=1}^{2} \pi^s_t.
\]

Since the speculator does not possess any superior information on the fundamental value of the foreign currency, she might find it useful to gain an informational advantage by using some noisy trading technology, \( \pi^s \), in period 1. As she does not have an incentive to preserve an informational advantage beyond period 2, no further noise trading will be considered.\(^7\)

Hence, suppose that in period 1 the speculator chooses a noise trading technology, \( \pi^s \), and places a random market order, \( b_1 \eta^1 \). Given that the total market order the dealer observes in period 1 is \( x_1 = a_1(f - s_0) + b_1 \eta^1 + \epsilon^1 \), the dealer sets the exchange rate in accordance to the weak-form efficiency condition. This is as in equation (1), for \( t = 1 \), where \( \lambda^1_1 \) is given by (4). Thus, considering that the speculator cannot predict her random market order before selecting the noise trading technology, the profits she expects from trading in the first period are given by:

\[
E[\pi^s_1|I^s_1] = -b_1^2 \lambda^1_1 \sigma^2_1.
\]

Moreover, consider that the expected fundamental value conditional on the information of the speculator at the end of period 1, \( s^*_1 \), is as in equation (5), for \( t = 1 \), where \( \lambda^*_1 \) is given by (6). Then, we can prove the following Lemma.

**Lemma 1** Suppose the speculator places a market order, \( b_1 \eta^1 \), according to a noise trading technology \( \pi^s \) in the first period. Then, her expected second period profits, conditional on her first period information set, are given by the following expression:

\[
E[\pi^*_2|I^*_1] = a_1 \Sigma_0 \left( \lambda^*_1 - \lambda^*_1 \right)^2 \left( \frac{1 - a_2 \lambda^*_2}{4 \lambda^*_2} \right),
\]

where \( \lambda^*_2 \) is the liquidity coefficient for period 2.

\(^7\)Instead, in a multi-period formulation the speculator could find it profitable to use a trading strategy which comprises a noise component in all, but the last, rounds of trading. Therefore, our conclusion that noise trading might be part of a profitable trading strategy would be strengthened.
Since \((\lambda_1^s - \lambda_1)\) and \(b_1^2 \lambda_1 \sigma_l^2\) are both increasing in \(b_1\), the speculator in period 1 faces a trade-off between second period profits and first period losses. Indeed, in order to acquire an informational advantage the speculator has to inject noise in the order flow. Given that in period 1 she does not have more information than the dealer and that this provides liquidity to the market with a transaction cost, the speculator faces a cost for her “informational investment”. The cost for this investment is increasing in the noise injected in the order flow, and so are the profits the speculator can obtain in the second round of trading. In fact, these are increasing in \(a_1 \Sigma_0 (\lambda_1^s - \lambda_1)\), which measures the informational advantage she acquires in the first period.

To establish if manipulation on the part of the speculator takes place, we need to see which noise trading technology she selects in period 1. This technology will be chosen maximizing the expected value of her total profits, \(\pi^s\), that is:

\[
b_1 = \arg \max \ E[\pi^s | I^s_1], \quad \text{where} \quad E[\pi^s(b_1) | I^s_1] = -b_1^2 \lambda_1 \sigma_l^2 + a_1 \Sigma_0 (\lambda_1^s - \lambda_1) \left( \frac{(1 - a_2 \lambda_2)^2}{4 \lambda_2} \right).
\]

(8)

Unfortunately, a simple analytical solution for this optimisation problem does not exist, since \(\lambda_1\) and \(\lambda_2\) depend on \(b_1\) in a complicated way, and unless we impose some restrictions on the parameters of the model, some numerical procedure will be necessary to find the optimal value of \(b_1\). Anyway, there is an important case in which an analytical solution exists.

### 3.2 Sporadic Central Bank Intervention

As we already suggested, central banks often intervene in the market for foreign exchange. In effect, sterilised intervention may be employed to signal changes in the monetary policy and condition market expectations and exchange rates.\(^8\) However, this kind of intervention is generally sporadic and concentrated in short periods of time, so that there always exists a spell of time between this intervention and the release of data on monetary aggregates. In other words, we refer to situations in which a sterilised operation is carried out a day before some announcement on the monetary growth. In this case, intervention operations by, say, the Bundesbank are carried out in the morning and early in the afternoon, while trading in the foreign exchange market continues around the clock.\(^9\) Thus, in our model the first period corresponds to that of the activity of the

\(^{8}\)See Dominguez and Frankel (1993) for a general discussion on foreign exchange policy and a detailed analysis of data on foreign exchange intervention. See also Vitale (1999) for a formal analysis of the signalling effect of sterilised intervention.

\(^{9}\)Dominguez (1997) presents evidence that central bank intervention is concentrated in specific periods of the day for the Fed, the Bundesbank and the Bank of Japan.
central bank, whilst the second refers to the rest of the day. We can then introduce the following
Assumption.

**Assumption 2** In the second round of trading the dealer receives market orders only from the
speculator and the liquidity traders, in that \( a_2 = 0 \).

Then, the following Proposition is proved in the Appendix.

**Proposition 2** Under the conditions of Assumption 2 there exists a unique linear Nash equilibrium
of the foreign exchange market with manipulation on the part of an uninformed speculator. The
manipulation mechanism of the speculator comprises the use of a noisy trading technology \( t \) in the
first period, and an informative market order in the second one. In equilibrium, the market orders
of the speculator and the exchange rates are as follows:

\[
x_1^s = b_1 \eta_1, \quad x_2^s = b_2 (s_1^s - s_1),
\]

\[
s_1 = s_0 + \lambda_1 x_1, \quad s_2 = s_1 + \lambda_2 x_2,
\]

where:

\[
b_1 = \left( \frac{a_1^2 \Sigma_0 + \sigma_t^2}{16a_1^2 \Sigma_0 + 15\sigma_t^2} \right)^{1/2}, \quad b_2 = \frac{4(a_1^2 \Sigma_0 + \sigma_t^2)}{a_1 \Sigma_0},
\]

\[
\lambda_1 = \frac{a_1 \Sigma_0 (16a_1^2 \Sigma_0 + 15\sigma_t^2)}{16(a_1^2 \Sigma_0 + \sigma_t^2)^2}, \quad \lambda_2 = \frac{a_1 \Sigma_0}{8(a_1^2 \Sigma_0 + \sigma_t^2)},
\]

\[
s_1^s = s_0 + \lambda_1^s y_1, \quad s_2^s = \frac{a_1 \Sigma_0}{a_1^2 \Sigma_0 + \sigma_t^2}.
\]

This Proposition suggests that foreign exchange intervention stimulates trading in the market for
foreign exchange, because a speculator enters in the market in order to acquire information from
the central bank activity and gain speculative profits. On turn, her action increases the volatility
of exchange rates. Consider, in fact, that the variance of the exchange rate conditional on \( f \) is
\( \lambda_1^2 \sigma_t^2 \) in period 1 and \( \lambda_2^2 \sigma_t^2 \) in period 2. These values are larger when \( b_1 > 0 \), that is when the
speculator enters in the market. In effect, central bank intervention is often accompanied by a rise
in the volume of transactions and the volatility of exchange rates and it is a least reassuring that
the present model captures this aspect of the functioning of the foreign exchange market. Notice,
also, that a similar conclusion is obtained by Madrigal (1996), as he finds that non-fundamental
speculation induces a rise in the volatility of prices.

To complete the analysis of this equilibrium consider that from the expressions for \( \lambda_1^s, \lambda_1 \) and \( b_1 \)
we find that the total expected profits of the speculator in equilibrium are positive and equal to:

\[
E[\pi^s | I_1^s] = \frac{a_1 \Sigma_0 \sigma_t^2}{16(a_1^2 \Sigma_0 + \sigma_t^2)}.
\]
4 Extensions

4.1 Optimal Central Bank Intervention

So far we have not specified how intervention operations on the part of the central bank are selected. However, the mechanism of selection of these operations will condition the choice of the uninformed speculator. In this respect, it is generally argued that central banks intervene mainly to smooth exchange rates, target their values and reduce market instability. While we could consider all these reasons for intervention, we are going to assume that the central bank just maximises its expected profits. In effect, during most days portfolio managers in the dealing room of a central bank conduct intervention operations with the simple objective of gaining profits, as any other trader.\footnote{Since sterilised intervention is potentially expensive, any specification of a loss function for the central bank should incorporate the cost of intervention and hence our results would not change dramatically if we considered other motives for the activity of the central bank.}

**Assumption 3** *The central bank enters in the foreign exchange market only in period 1, placing a market order, $x^b_1$, with the intention of maximising its expected profits.*

Since the central bank is fully rational and that it anticipates the presence of the speculator, its optimal market order in period 1, $x^b_1$, is given by the following expression:

$$x^b_1 = a_1(f - s_0), \quad \text{where} \quad a_1 = \left(\frac{\sigma^2}{\Sigma 0}\right)^{1/2} \quad \text{and} \quad \sigma^2 = \sigma^2_1 (1 + b^2_1).$$

Then, it is just a question of tedious algebra to prove the following Proposition.
Proposition 3  Under the conditions of Assumption 3 there exists a unique linear Nash equilibrium of the foreign exchange market with manipulation on the part of an uninformed speculator and optimal central bank intervention. The equilibrium is as described in Proposition 2 with the following trading intensities of the central bank and the speculator:

\[ b_1 = \left( \frac{2}{\sqrt{257} + 15} \right)^{1/2}, \quad b_2 = \frac{\sqrt{257} + 17}{(\sqrt{257} + 1)^{1/2}} \left( \frac{\sigma_t^2}{\Sigma_0} \right)^{1/2}, \quad a_1 = \left( \frac{\sqrt{257} + 1}{16} \frac{\sigma_t^2}{\Sigma_0} \right)^{1/2}, \]

(15)

while the coefficients \( \lambda_1, \lambda_2 \) and \( \lambda_1^* \) are:

\[ \lambda_1 = \frac{1}{2} \left( \frac{16}{\sqrt{257} + 1} \frac{\Sigma_0}{\sigma_t^2} \right)^{1/2}, \quad \lambda_2 = \frac{1}{2} \left( \frac{\sqrt{257} + 1}{\sqrt{257} + 17} \right)^{1/2} \left( \frac{\Sigma_0}{\sigma_t^2} \right)^{1/2}, \quad \lambda_1^* = 8\lambda_2. \]

(16)

As we have already claimed, when a central bank, say the Bundesbank, is active in the foreign exchange market a dealer, say Deutsche Bank, will stand as market leader in the formulation of market expectations and the setting of exchange rates. Then, according to our interpretation, a mechanism of manipulation is possible in the foreign exchange market, in that a speculator can anonymously trade with the market leader and manipulate his expectations.\(^{11}\)

Now, in our model the unconditional variance of the total order flow in period 1 is \( a_1 \Sigma_0 + \sigma_1^2 \), while that of the market order of the speculator is simply \( b_1^2 \sigma_t^2 \). Inserting in the ratio between the latter and the former the equilibrium values of \( a_1 \) and \( b_1 \) given in Proposition 3, one finds that the share of volume of trading due to manipulation is of the order of 3.27%. Suppose, thus, that in a morning of activity of the Bundesbank, Deutsche Bank completes on average transactions for $1 billion.\(^{12}\)

In this case, our model would suggest that a speculator could manipulate his expectations and gain speculative profits by placing market orders for a total of $32.7 million. This figure indicates a dimension of manipulation of limited scale, accessible to most large fund managers in the foreign exchange market.

Despite its scale is limited, manipulative trading has important consequences for the market characteristics. To fully appreciate them, let us consider the benchmark case in which the uninformed speculator does not enter in the foreign exchange market. In this case the market equilibrium in period 1 corresponds to the static version of the original model discussed by Kyle, so that the liquidity coefficient charged by the dealer, \( \lambda_1^* \), the trading intensity of the central bank, \( a_1^* \), and its

\(^{11}\)Moreover, some evidence by LeBaron (1996), Neely and Weller (1997), and Szakmary and Mathur (1997) shows that technical traders gain significant profits during days of central bank intervention. This is consistent with our interpretation of the possibility of manipulation open to noise traders.

\(^{12}\)This figure is not distant from the average daily volume of trading for foreign exchange dealers as reported by Lyons (1995).
unconditional profits, $E[\pi^b]$ are equal to:

$$\lambda_1^* = \frac{1}{2} \left( \frac{\Sigma_0}{\sigma_1^2} \right)^{1/2}, \quad a_1^* = \left( \frac{\sigma_1^2}{\Sigma_0} \right)^{1/2}, \quad E[\pi^b] = \frac{1}{2} (\sigma_1^2 \Sigma_0)^{1/2}.$$  

Comparing the values of $\lambda_1$ with $\lambda_1^*$, we see that price manipulation reduces transaction costs by 3.07% in the first period, increasing the liquidity of the market when the central bank is active. Notice, however, that the trading intensity of the central bank, $a_1$, rises too, in such a way that the informativeness of the flow of orders remains unchanged. In fact, in both cases at the end of period 1 the residual uncertainty of the market maker on the fundamental value, measured by the conditional variance $\Sigma_1$, is the same. In particular, the central bank will find it optimal to consume half of its informational advantage, and so $\Sigma_1 = \Sigma_0/2$. Since more noise is present in the order flow the central bank will gain larger expected profits. These will move from $E[\pi^b]$ to $[\sigma_1^2 \Sigma_0 (\sqrt{257} + 1)/64]^{1/2}$ with a rise of 3.17%.

Notice that this differs from the conclusions of Madrigal (1996). In fact, he finds that non-fundamental speculation reduces the expected profits of the agent who possesses fundamental information. Then, since the incentive to acquire fundamental information is hampered the efficiency of the market may fall. Obviously, this difference is induced by the particular institutional arrangement we have introduced when assuming that the central bank is active only in the first period of trading.

By inserting noise in the order flow, the speculator gains an informational advantage. In particular, while the residual uncertainty of the market maker at the end of period 1 is $\Sigma_1 = \Sigma_0/2$, that of the speculator is $\Sigma_s^1 = [16/(\sqrt{257} + 17)] \Sigma_0$, so that she acquires a small informational advantage with respect to the dealer, of the order of 3.12%. In period 2, she will exploit it placing an informative market order with the dealer. This will make the order flow informative in the second period as well, raising, even if marginally, transaction costs and smoothing liquidity across the two periods the market is open.

On the other hand, the speculator will raise the efficiency of the market. In fact the conditional variance of $f$ given the information of the dealer at the end of period 2, $\Sigma_2$, is equal to

$$\Sigma_2 = \frac{1}{2} \left[ 1 - \frac{16}{(\sqrt{257} + 17)(\sqrt{257} + 15)} \right] \Sigma_0,$$

with a reduction with respect to the corresponding value for period 1, $\Sigma_1$, of the order of 1.56%. Notice how comparing $\Sigma_2$ with $\Sigma_s^1$ one can prove that, as the central bank, the speculator finds it convenient to consume half of her informational advantage.\footnote{In fact, it is not difficult to prove that $\Sigma_1 - \Sigma_s^1 = 2(\Sigma_1 - \Sigma_2)$.} Since such an informational advantage

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is small she will make small profits. In fact, the value of her unconditional expected profits, $E[\pi^s]$, is equal to $[\sigma_l^2 \Sigma_0 \left(\sqrt{257} + 1\right)/16]^{1/2}/(\sqrt{257} + 17)$ and hence smaller than 1/8 of that for the central bank.

4.2 Continuous Central Bank Intervention

Despite the presence of central banks in the foreign exchange market is sporadic, it might be interesting to establish if manipulation takes place when sterilised intervention is continuous. As we said, when $a_2$ is positive we are not able to provide a closed form solution for the optimisation problem of the speculator and a numerical procedure will be required. Nevertheless, general qualitative results may be obtained studying equation (8). In particular, the following Lemma provides a useful intuitive result.

**Lemma 2** For any noise trading technology selected by the speculator in period 1, her expected profits are decreasing in $a_2$.

In fact, an increase in the intensity of trading of the central bank, $a_2$, will force a reduction of the market order of the speculator in the second round of trading with a contraction of her expected profits. This Lemma immediately leads to the following important Proposition.

**Proposition 4** For any choice of $a_1$, there exists a minimum value $a_2$ such that for $a_2 \geq a_2$, $E[\pi^s | I^*_1]$ is negative for any noise trading technology $t$. As a consequence, the set of points $(a_1, a_2)$ such that no manipulation takes place is non-empty.

Indeed, as suggested by this Proposition, an aggressive trading rule of the central bank can always force the speculator out of the market. As an increase in $a_2$ reduces the value of the “informational investment”, the speculator will not find it convenient to meet the initial cost for its acquisition and there will not be any manipulation.

As an explicative example consider Figure 1. Here, we present the graph of the expected profits of the speculator for two different trading rules of the central bank when $\sigma_l = \Sigma_0 = 1$. In the left panel $a_1 = 10 a_2$, while in the right one $a_1 = a_2/2$. As it is clear from the two panels, only when the central bank trades much less aggressively in the second period than in the first one, the speculator can expect profits when entering in the market. For $a_1 < a_2$, instead, the profits she can gain in the second round of trading cannot cover the losses she incurs in the first one and the speculator does not enter in the market at all. In synthesis, an active central bank can severely limit manipulation of the kind discussed here.
In addition, notice that the values of $a_1$ and $a_2$ used in the right panel corresponds to those of the sequential equilibrium of the original Kyle’s model, in which an insider maximises her expected profits when only two auctions are called and no manipulation takes place. Unfortunately, in view of its analytical complexity, we cannot formally prove that when the values of $a_1$ and $a_2$ are given by Kyle’s sequential equilibrium an uninformed trader will never enter in the market in order to manipulate expectations and exchange rates. However, a simple numerical analysis showed that this result withholds for all parametric configurations we considered. This conclusion has two important implications. First, a profit maximising central bank, which can trade around the clock, will not let an uninformed speculator manipulate market expectations and exchange rates. Second, Kyle’s sequential equilibrium is robust with respect to the mechanism of price manipulation we have investigated.

4.3 Multiple Speculators

Noise trading is a free good. This implies that when one uninformed trader finds it convenient to inject noise in a dealership market, others would like to follow. Suppose then that in the market there exist $M$ speculators, who place in period 1 uncorrelated random market orders. In particular, we consider the possibility that all speculators, $i = 1, 2 \ldots, M$, place market orders $x_{1i}$, given by $b_{1i} \eta_{1i}$, where $\eta_{1i} \sim N(0, \sigma^2_1)$ and $\eta_{1i} \perp \eta_{1j}$ for $i \neq j$. The total order flow in period 1 is then:

$$x_1 = a_1 (f - s_0) + \sum_{i=1}^{M} b_{1i} \eta_{1i} + \epsilon_1.$$  

Now, the market maker fixes the spot rate in period 1 equal to:

$$s_1 = s_0 + \lambda_1 x_1,$$

where

$$\lambda_1 = \frac{a_1 \Sigma_0}{a_1^2 \Sigma_0 + \sigma^2_1}, \quad \text{and} \quad \sigma^2_1 = \left[ 1 + \sum_{i=1}^{M} b_{1i}^2 \right] \sigma^2_l.$$  

Any speculator, $i$, will then observe her private signal, $y_{1i} = x_1 - b_{1i} \eta_{1i}$, so that applying the projection theorem she will form her own expectation of the fundamental value:

$$s_{1i} = s_0 + \lambda_{1i} y_{1i}, \quad \text{where}$$

$$\lambda_{1i} = \frac{a_1 \Sigma_0}{a_1^2 \Sigma_0 + \sigma^2_{si}}, \quad \text{and} \quad \sigma^2_{si} = \left[ 1 + \sum_{j \neq i} b_{1j}^2 \right] \sigma^2_l.$$  

However, we will consider only symmetric equilibria, so that we will have $b_{1i} = b_1$. In particular, to find a symmetric equilibrium, assume that for $j \neq i$ $b_{1i} = b_1$. Then, under Assumption 2, in period
the optimal market order of speculator $i$ is:

$$x_{2i}^* = b_2^i(s_{si}^i - s_1), \quad \text{where} \quad b_2^i = \frac{1}{\lambda_2 (2 + (M - 1)\phi_2^i)}, \quad \phi_2^i = \frac{a_1^2 \Sigma_0 + \sigma_{pi}^2}{a_1^2 \Sigma_0 + \bar{\sigma}_{si}^2}.$$

$$\sigma_{pi}^2 = [1 + (M - 2)b_1^2]\sigma_l^2 \quad \text{and} \quad \sigma_{si}^2 = [1 + (M - 2)b_1^2 + b_2^i]\sigma_l^2.$$ Here $\phi_2^i$ is the coefficient of the projection of the signal of speculator $j$, $s_{sj}^j - s_1$, onto that of speculator $i$, $s_{si}^i - s_1$, while $\lambda_2$ is the usual liquidity coefficient for period 2, as the pricing rule of the market maker is:

$$s_2 = s_1 + \lambda_2 x_2.$$

Then, speculator $i$ will select her initial random market order by maximising her expected total profits given by the following expression:

$$E[\pi_{si}^i | I_{si}^i] = -\lambda_1 b_1^2 \sigma_l^2 + \frac{a_1 \Sigma_0 (\lambda_{si}^i - \lambda_1)}{\lambda_2 (2 + (M - 1)\phi_2^i)^2}. \quad (17)$$

Finally, the condition $b_1^i = b_1$ will be imposed to find a symmetric equilibrium. Unfortunately a simple analytical solution is not available for the maximisation problem of speculator $i$, so that a numerical procedure will be required.

In Table 1 we report the equilibrium values of $b_1$ for different numbers of speculators, when $a_1 = 1$, $\Sigma_0 = 1$, $\sigma_l^2 = 1$. Clearly, an increase in $M$ severely reduces the intensity of the noisy market order of single speculators, $b_1$, alongside their expected profits, $E[\pi_{si}^i | I_{si}^i]$, and the total volume of noise added to the order flow of the market maker by the entire population of speculators, $\sqrt{Mb_1}\sigma_l$. In the limit, as the number of speculators rises to infinite, the incentive to introduce noise in the order flow is totally eliminated both individually and collectively.

To explain these conclusions consider that an increase in the number of speculators presents three effects. Firstly, for a given individual trading intensity $b_1$, more speculators will augment the liquidity of the market in period 1. Thus, as $\lambda_1$ falls, the cost of injecting noise for speculator $i$ reduces. This should induce her to rise $b_1$. Secondly, as more speculators inject noise in the order flow, speculator $i$ receives a less informative signal in period 1, $y_{1i}^i = x_1 - b_1 \eta_{1i}^1$, and her informational advantage with respect to the dealer, $a_1 \Sigma_0 (\lambda_{si}^i - \lambda_1)$, deteriorates. Thirdly, the uninformed speculators conduct a Cournot competition in period 2, so given any level of her informational advantage in period 2, speculator $i$ will trade less aggressively when $M$ rises. The last two effects clearly induce her to reduce $b_1$.

Our numerical analysis indicates that the latter effects dominate the former and more speculators imply less manipulation. This suggests that free entrance to the foreign exchange market limits

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14The proof that speculator $i$ maximises the expected profits given in equation (17) follows very closely that for a single uninformed speculator discussed in Section 3. Details can be obtained from the author on request.

15Numerical analysis shows that our choice of $a_1$, $\Sigma_0$ and $\sigma_l^2$ is inconsequential for these qualitative results.
the use of speculative noise trading. However, notice that trading activity is expensive in most securities markets. In particular, it is fairly costly to open a foreign exchange dealing room and therefore, if we introduce a cost for the entrance into the activity of manipulation, we can endogenise the number of speculators, $M$. Clearly, larger markets, i.e. markets where the volume of liquidity trading $\sigma_l^2$ is larger, will have a larger community of uninformed speculators, as the expected profits from speculative noise trading, $E[\pi^{un}|I_1^{un}]$, are increasing with $\sigma_l^2$.

5 Concluding Remarks

In this paper we have considered a possible reason which may induce uninformed speculators to trade on the basis of irrelevant information in the market for foreign exchange. We suggest that noise trading may be used to manipulate expectations and exchange rates in order to gain an informational advantage and hence a profit opportunity. To investigate this opportunity, we have developed a two auction model of the foreign exchange market, in which uninformed speculators can exploit the impossibility of the market maker to distinguish between informed and uninformed clients to manipulate his expectations and the exchange rates and hence gain speculative profits. The main results of our analysis are as follows.

Injecting noise in the order flow of the dealer, a rational uninformed speculator interferes with the learning process of the market maker and determines his degree of uncertainty on the fundamental value of a foreign currency. The possibility of reducing the resiliency of the market, that is the speed of convergence of the exchange to its fundamental value, permits the speculator to gain and preserve an informational advantage with respect to the dealer.

The acquisition of this informational advantage corresponds to an “informational investment”, because the dealer provides liquidity to the market at a cost and hence placing random market orders is expensive. On the other hand, the informational advantage yields profits from future informed trading. The solution of the trade-off between these costs and profits determines the optimal volume of speculative noise trading.

The equilibrium of the foreign exchange market with sporadic central bank intervention clearly indicates that speculative noise trading may be part of a profitable trading strategy in the foreign exchange market. In effect, the practice of giving spurious signals to other market participants to condition their expectations is common among professional traders. Investigating the market for foreign exchange, Lyons (1995) finds that a large component of the order flow of a dealer does not contain information. Because most transactions in the foreign exchange market are between professional traders, we can interpret this as evidence of attempts of manipulation.
However, our analysis also outlines the limits that exist to this kind of speculative activity. In particular, when central bank intervention is continuous, the scope for manipulation can be severely reduced, because the monetary authorities have the faculty of excluding uninformed speculators from the market. This also suggests that in other dealer markets, in which informed agents trade continuously, the kind of manipulation discussed here is less likely to emerge. Finally, since noise trading is a free good, competition among uninformed speculators will reduce the incentive they have to trade on pure noise.

6 Appendix

Proof of Proposition 1.
Consider the inequality $\Sigma_t > \Sigma_t^s$, where $t = 1, 2$. From the definition of the conditional variances for the market maker and the speculator we have that:

$$\Sigma_t = \frac{\Sigma_{t-1}}{1 + \frac{a_t^2}{\sigma_t^2} \Sigma_{t-1}}, \quad \Sigma_t^s = \frac{\Sigma_{t-1}^s}{1 + \frac{a_t^2}{\sigma_t^2} \Sigma_{t-1}^s}.$$  

So that the inequality among the variances is equivalent to:

$$\Sigma_{t-1} - \Sigma_{t-1}^s + a_t^2 \Sigma_{t-1} \Sigma_{t-1}^s \left( \frac{1}{\sigma_t^2} - \frac{1}{\sigma_t^2} \right) > 0.$$  

For $t = 1$, note that $\Sigma_0^s = \Sigma_0$. Then, since for $b_1 > 0 \sigma_1^2 > \sigma_1^2$, $\Sigma_1 > \Sigma_1^s$. For $t = 2$ the proof that $\Sigma_t > \Sigma_t^s$ is immediate. In fact, if $\Sigma_1 > \Sigma_1^s$ we have that $\Sigma_2 > \Sigma_2^s$ for any value of $b_2$. \Box

Proof of Lemma 1.
We start this proof by analyzing the game which the dealer and the speculator play in the second period. Suppose the expected fundamental value of the speculator at the end of period 1 is $s_1^s$ and that the pricing rule of the market maker for the second period is:

$$s_2 = s_1 + \lambda_2 x_2,$$  

where $s_t$ is the exchange rate in period $t$ and $x_2$ is the total order flow received by the market maker in period 2. Hence, it is immediate to see that:

$$x_2^s = b_2 (s_1^s - s_1), \quad \text{where}$$

$$b_2 = \frac{1 - a_2 \lambda_2}{2 \lambda_2}.$$  

Then, it is simple to verify that in the second period the expected profits of the speculator are:

$$E[\pi_2^s | I_2^s] = \frac{[(1 - a_2 \lambda_2)(s_1^s - s_1)]^2}{4 \lambda_2}.$$  

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Thus, the expected value of these profits, conditional on the speculator information set at the beginning of period 1, is given by:

\[ E[\pi_s^2|I^*_s] = \frac{(1-a_2\lambda_2)^2}{4\lambda_2} E[(s_1^s-s_1)^2|I^*_s]. \]  

(20)

In order to evaluate this expectation, consider that in period 1 if the speculator places a random market order \( b_1\eta_1 \), the dealer receives the following total market order, \( x_1 = a_1(f-s_0) + b_1\eta_1 + \epsilon_1 \). He can use this observation of the order flow to form an expectation of the fundamental value, \( f \):

\[ s_1 = s_0 + \lambda_1 x_1, \quad \text{where} \quad \lambda_1 = \frac{a_1\Sigma_0}{a_1^2\Sigma_0 + \sigma^2_1}. \]  

(21)

Observing the exchange rate at the end of the first round of trading the speculator can recover the value of the total market order, \( x_1 \), from equation (21). Thus, she is able to form a more efficient expectation of \( f \):

\[ s_1^s = s_0 + \lambda_1^s y_1, \quad \text{where} \quad \lambda_1^s = \frac{a_1\Sigma_0}{a_1^2\Sigma_0 + \sigma^2_1}, \]  

(22)

and \( y_1 = x_1 - b_1\eta_1 \). In period 2 the market maker observes a new value of the order flow. Considering the expressions for \( x_2 \) and for \( s_1^s \), this observation of the order flow becomes:

\[ x_2 = a_2(f-s_1) + a_1b_2\lambda_1^s f - b_2s_1 + b_2\lambda_1^s \epsilon_1 + \epsilon_2. \]

Then, applying the projection theorem one obtains that the exchange rate in the second round of trading is given by equation (18), since the expected value of \( x_2 \) given the information set of the dealer at time 1 is null. In other words, the strategies of the two agents (in equations (18) and (19)) are mutually consistent.

Considering the expressions of \( s_1, s_1^s \) and those of \( \Sigma_0 \) and \( \lambda_1 \), we can derive the following expectations:

\[ E[(s_1^s)^2|I^*_s] = a_1\Sigma_0\lambda_1^s, \quad E[s_1^s|I^*_s] = a_1\Sigma_0\lambda_1, \quad E[s_1^ss_1|I^*_s] = a_1\Sigma_0\lambda_1. \]

Plugging these expectations in the right hand side of equation (20) one can obtain the following expression:

\[ E[\pi_s^2|I^*_s] = a_1\Sigma_0(\lambda_1^s - \lambda_1) \left( \frac{(1-a_2\lambda_2)^2}{4\lambda_2} \right). \]  

\[ \square \]

**Proof of Proposition 2.**

To prove the Proposition we just need to show that the speculator finds it useful to employ a noise trading technology \( t \) in period 1 and then recoup the initial losses this generates in the second round of trading. Note that only unpredictable market orders in period 1 are useful, because otherwise
the predictable part of \( x^*_1 \) would be filtered out from the total order flow, without affecting its informativeness, but increasing the expected losses of the speculator in period 1. Furthermore, considering that \( a_2 = 0 \) the speculator maximizes with respect to \( b_1 \) the following expression:

\[
E[\pi^s|I^s_1] = -b_1^2 \lambda_1 \sigma_i^2 + \frac{1}{4\lambda_2} a_1 \Sigma_0 (\lambda^*_1 - \lambda_1).
\]

In order to find \( \lambda_2 \) we need to investigate the determination of the liquidity coefficient in the second period. Using the projection theorem once again, we find that:

\[
\lambda_2 = \frac{a_1 \Sigma_0 (\lambda^*_1 - \lambda_1) b_2}{a_1 \Sigma_0 (\lambda^*_1 - \lambda_1) b_2^2 + \sigma_i^2}, \tag{23}
\]

Since \( b_2 = 1/(2\lambda_2) \), it easy to prove that equation (23) becomes:

\[
\lambda_2 = \frac{\sqrt{a_1 \Sigma_0 (\lambda^*_1 - \lambda_1)}}{2\lambda_1}. \tag{24}
\]

It is then immediate to insert \( \lambda_2 \) in the expected second period profits and verify that the speculator maximises the following expression:

\[
E[\pi^s(b_1)|I^s_1] = \left\{ -b_2^2 \lambda_1 \sigma_i^2 + \frac{\sigma_i}{2} \sqrt{a_1 \Sigma_0 (\lambda^*_1 - \lambda_1)} \right\}.
\]

Let us take the first derivative of \( E[\pi^s(b_1)|I^s_1] \) with respect to \( b_1 \). Then, considering that

\[
\frac{\partial b_1^2 \lambda_1 \sigma_i^2}{\partial b_1} = 2b_1 \lambda_1 \sigma_i^2 \left( \frac{a_1^2 \Sigma_0 + \sigma_i^2}{a_1^2 \Sigma_0 + \sigma_i^2} \right),
\]

\[
\frac{\partial \sigma_i \sqrt{a_1 \Sigma_0 (\lambda^*_1 - \lambda_1)}}{\partial b_1} = \frac{b_1 \lambda_1 \sigma_i^2}{a_1^2 \Sigma_0 + \sigma_i^2} \frac{a_1 \sigma_i \Sigma_0}{\sqrt{a_1 \Sigma_0 (\lambda^*_1 - \lambda_1)}},
\]

we obtain that:

\[
\frac{\partial E[\pi^s(b_1)|I^s_1]}{\partial b_1} = \frac{2b_1 \lambda_1 \sigma_i^2}{a_1^2 \Sigma_0 + \sigma_i^2} \left[ \frac{a_1 \sigma_i \Sigma_0}{4\sqrt{a_1 \Sigma_0 (\lambda^*_1 - \lambda_1)}} - (a_1^2 \Sigma_0 + \sigma_i^2) \right].
\]

A root of the first derivative of \( E[\pi^s(b_1)|I^s_1] \) corresponds to a root of the following equation:

\[
\frac{a_1 \sigma_i^2 \Sigma_0}{\lambda^*_1 - \lambda_1} = 16(a_1^2 \Sigma_0 + \sigma_i^2)^2. \tag{25}
\]

Now, considering that

\[
\lambda^*_1 - \lambda_1 = \frac{a_1 b_2^2 \sigma_i^2 \Sigma_0}{(a_1^2 \Sigma_0 + \sigma_i^2) (a_1^2 \Sigma_0 + \sigma_i^2)},
\]

a root of equation (25) corresponds to a root of the following one:

\[
(a_1^2 \Sigma_0 + \sigma_i^2) = 16b_2^2 (a_1^2 \Sigma_0 + \sigma_i^2).
\]
This quadratic equation possesses two distinct roots, symmetric around zero. We can take the positive one. We do not need to check for the second derivative, because for $b_1 \downarrow 0$ the negative part of the derivative goes to zero faster than the positive one, so that we have a positive maximum. Thus, in the first round of trading the speculator employs a noise trading technology $t$, where:

$$b_1 = \left( \frac{a_1^2 \Sigma_0 + \sigma_i^2}{16a_1^2 \Sigma_0 + 15\sigma_i^2} \right)^{1/2}. $$

Finally, following forward the solution of the model already sketched completes the proof and gives a full characterization of the equilibrium of the market. In fact, the exchange rate set in the first round of trading is given by equation (21),

$$s_1 = s_0 + \lambda_1 x_1,$$

where, plugging the expression of $b_1$ in $\lambda_1$, the liquidity coefficient is:

$$\lambda_1 = \frac{a_1 \Sigma_0 (16a_1^2 \Sigma_0 + 15\sigma_i^2)}{16(a_1^2 \Sigma_0 + \sigma_i^2)^2}. $$

Then, plugging equation (25) in equation (24) we find that the liquidity coefficient in period 2 is given by the following expression:

$$\lambda_2 = \frac{a_1 \Sigma_0}{8(a_1^2 \Sigma_0 + \sigma_i^2)}. $$

Thus, in the second period, considering $\lambda_2$, the speculator places the following market order:

$$x_2^s = b_2(s_1^s - s_1),$$

where

$$b_2 = \frac{4(a_1^2 \Sigma_0 + \sigma_i^2)}{a_1 \Sigma_0},$$

and $s_1^s$ is given by equation (22). Finally, the dealer sets the exchange rate according to equation (18):

$$s_2 = s_1 + \lambda_2 x_2. \Box$$

**Proof of Proposition 3.**

To prove this Proposition we just need to find the mutually consistent values for the trading intensities $a_1$ and $b_1$. Inserting the expression for the former given in equation (14) into that for the latter in equation (11), one finds the equilibrium value of $b_1$ in Proposition 3. It is then straightforward to find the value for $a_1$ and the other coefficients characterising the market equilibrium. \Box

**Proof of Lemma 2.**

To prove this Lemma we need to show that $(1 - a_2 \lambda_2)^2 / 4 \lambda_2$ is decreasing in $a_2$. Therefore, we need
to calculate the liquidity coefficient $\lambda_2$. In accordance with the projection theorem:

$$ s_2 = s_1 + \lambda_2 x_2, \quad \text{where} $$

$$ \lambda_2 = \frac{\text{cov}(f, x_2 | I_1^m)}{\text{var}(x_2 | I_1^m)}, $$

in that $E[x_2 | I_1^m] = 0$. Calculating these variance and covariance we find that:

$$ \lambda_2 = \frac{(a_1 \lambda^*_1 b_2 + a_2)(1 - a_1 \lambda_1)\Sigma_0 - \lambda_1 \lambda^*_1 \sigma^2_1 b_2}{(a_1 \lambda^*_1 b_2 + a_2)^2(1 - a_1 \lambda_1)\Sigma_0 + (\lambda^*_1)^2(1 - \lambda^*_1)\sigma^2_1 b_2^2 - (a_1 \lambda^*_1 b_2 + a_2)\lambda_1 \lambda^*_1 \sigma^2_1 + \sigma^2_t}, $$

where $\lambda_1^* = \sigma^2_t \lambda_1 / a_1 \Sigma_0$. Considering that $b_2 = (1 - a_2 \lambda_2)/2 \lambda_2$ we have that:

$$ \lambda_2 = b_2 = \frac{1}{a_2 + 2b_2}. $$

Substituting this expression in equation (26) and rearranging gives the following equation in $b_2$:

$$ Ab_2^2 + Bb_2 + C = 0, \quad \text{where} $$

$$ A = a_1 \Sigma_0 (\lambda^*_1 - \lambda_1) > 0, $$

$$ B = a_2 \Sigma_0 [\lambda^*_1 + (1 - a_1 \lambda_1)] > 0, $$

$$ C = -\sigma^2_t, $$

and $\lambda_t^* = \sigma^2_t / a_1 \Sigma_0$. Then, taking the positive root of this equation we obtain:

$$ b_2 = \frac{\Delta - B}{2A}, $$

$$ \lambda_2 = \frac{A}{a_2 A - B + \Delta}, $$

where $\Delta = \sqrt{B^2 - 4AC}$. From these two equations it is immediate to check that: $(1 - a_2 \lambda_2) > 0$, and that:

$$ \frac{\partial (1 - a_2 \lambda_2)}{\partial a_2} = \frac{A (B^2 - \Delta^2)}{\Delta (a_2 A - B + \Delta)^2} < 0, $$

$$ \frac{\partial b_2}{\partial a_2} = \frac{B (B - \Delta)}{2a_2 A \Delta} < 0. $$

Since $(1 - a_2 \lambda_2)^2 / 4 \lambda_2 = b_2 (1 - a_2 \lambda_2)/2$, an increase in $a_2$ reduces the expected second period profits of the speculator, given the noise trading technology $t$. $\square$

**Proof of Proposition 4.**

This proof is straightforward. Consider the expression for $b_2$ in equation (27): for $a_2 \uparrow \infty \Delta \downarrow B$. Therefore, $b_2 \downarrow 0$ and so do the expected second period profits of the speculator for any choice of the noise trading technology $t$. $\square$
References


| Number of Speculators | $b_1$     | $\sqrt{M}b_1$ | $E[\pi^i|I_1^i]$ |
|-----------------------|-----------|---------------|------------------|
| 1                     | 0.25400025| 0.25400025    | 0.03125000       |
| 2                     | 0.12667012| 0.17913861    | 0.00256218       |
| 5                     | 0.03002898| 0.06714685    | 0.00004976       |
| 10                    | 0.00863711| 0.02731291    | 0.0000196        |
| 20                    | 0.00231457| 0.01038187    | 0.00000007       |
| 50                    | 0.00038828| 0.00274554    | 0.00000000       |

Notice: $a_1 = 1$, $\Sigma_0 = 1$, $\sigma_1^2 = 1.$
Figure 1: Expected Profits of the Speculator as Functions of $b_1$

The continuous line refers to the total expected profits, while the dotted ones refer to the expected first period losses and second period profits. $\sum_0 = \sigma_1 = 1.$