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1 INTRODUCTION

How trading arrangements within the foreign exchange market might affect the behavior of prices and allocative efficiency is largely unexplored territory. In a recent survey article, Flood (1991) stresses how little is known about basic features of the FX market. For example, why is the magnitude of inter-dealer trading (80% of total volume) so great? Why is half of that trading intermediated by brokers?

Of the few studies of FX market institutional arrangements, Grossman and Zhou (1991) study the impact of stop-loss trading rules typically used by FX dealers on optimal portfolio strategies of individual traders. Krugman and Miller (1993) adduce various consequences of such rules for the behavior of the FX market as a whole, arguing that they may provoke market crashes which could in turn provide a justification for commonly-observed intervention behavior by central banks like target zones. Bossaerts and Hillion (1991) examine the implications of dealer behaviour for unbiasedness tests in FX markets.

Finally, an interesting series of papers by Lyons (1992,1993a,1993b) directly analyzes the microstructure of the FX market. Lyons concentrates particularly on the role of brokers, viewing them as a means by which order flow information is aggregated and then disseminated among dealers. While this is certainly a fruitful line of research, it is not clear in such models why individual dealers do not have an incentive to deal directly rather than through brokers. Also, conversations with traders suggested to us that brokers are primarily important because of the efficient access they provide to large numbers of other market participants.

For these reasons, the analysis of FX trading that we attempt in this paper has a somewhat different focus from that of Lyons. We concentrate on the consequences for efficiency and exchange rate behavior of the market’s decentralized nature, i.e., the fact that dealers are ignorant of the order flow of other market-makers. Inter-bank trading is modelled as a means by which market-makers “sell” each other information about their transactions with outside customers. We show that, under these assumptions, decentralized market arrangements are privately efficient for the group of market-makers.

The reason that a decentralized market works well in this case is that it allows market-makers to capture, through inter-dealer trade, the informational rents associated with receiving outside orders and hence gives them an incentive to adjust their spreads optimally to maximize those
rents. If market-makers are only able to transact with a fraction of other dealers between customer orders, a centralized market in which order flow information is freely and instantaneously available may be preferable to decentralized arrangements. Although incentives to adjust bid-ask spreads efficiently are diluted in a centralized market, at least dealers can observe customer orders and rationally update their subjective probabilities.

An important aspect of the relative efficiency of different market arrangements is their robustness to extreme informational asymmetries. Glosten and Milgrom (1987) discuss the market crashes that may occur when dealers suspect that large numbers of informed agents are present. During such crashes, volume dries up as spreads widen and the informativeness of prices is lost. One of our more interesting findings is that such crashes happen much less in decentralized than in centralized markets such as those studied by Glosten and Milgrom (1987). The reason is that dealers have an incentive to maintain at least some turnover in order to elicit information that they can use in future trading.

As well as supplying efficiency results, our analysis sheds light on the implications of decentralized markets for the time series behavior of exchange rates. Most notably, we find, first, that the usual martingale properties of prices are absent, in that, in the decentralized market, the bid and ask on average increase and decline respectively as trades reveal information. Second, the unconditional variance of changes in bid and ask quotes is greater in the decentralized than in the centralized market. We also prove a series of comparative static results, demonstrating that, as one might expect, in the decentralized market spreads widen as the proportion of informed traders increases, while the bid price is increasing in the probability of higher exchange rate values.

Previous theoretical work on decentralized markets has been quite limited although recently several authors have begun to address the issue. Biais (1993) studies the effects of deviations from transparency in an inventory model of microstructure. Our approach, concentrating as it does on informational asymmetries, may be viewed as complimentary to Biais work. An interesting recent paper by Neuberger, Naik and Viswananthan (1993) examines the impact of trade publication delays on price formation in the London Stock Exchange. Although quite different from ours in modelling approach\(^1\), they emphasize as we do the informativeness of customer trades.

The structure of our paper is as follows. In Section 2, we set out the model, studying first a static model of dealing with informed and uninformed outside customers and then showing how this fits into a more complicated dynamic framework with two periods of customer trading.
separated by a period of inter-bank transactions. Section 3 describes the results we obtain with the model. These include results on efficiency, statistics of bid and ask prices, comparative statics for dealer prices and informational rents, and a result on inter-dealer market volume. An Appendix provides a complete account of the proofs of the various lemmas and propositions.

2 THE MODEL

2.1 Basic Assumptions

Suppose that there are n identical dealers and four periods, denoted 0,1,2,3. In period 0, each dealer selects a bid and ask and then trades with a customer if one presents himself. In period 1, dealers may trade among each other if they so wish. In period 2, dealers trade again with customers if there are any. In period 3, all uncertainty is resolved.

Assume for simplicity that all agents are risk neutral and that the interest rate is zero\(^2\). The structure of trading and of the stochastic order flow from customers is as follows (see Figure 1 for a summary). The value of the exchange rate in period 3 is the realization of a random variable, \(z\). Let the unconditional distribution of \(z\) be binomial in that \(z\) takes the values 1 and -1 with probabilities \(q\) and \((1 - q)\).

In period 0, a single customer arrives and is allocated with probability \(1/n\) to a given dealer. With probabilities \(\alpha\) and \(1 - \alpha\) respectively the trader is either informed or a liquidity-trader. Informed traders transact if and only if their observation of the true value, \(z\), exceeds the ask quoted by the dealer, \(s_A\), (in which case they buy) or falls below the bid price, \(s_B\), (in which case they sell). We suppose that liquidity-traders buy, sell or do not trade with probabilities, \((1 - s_A)/2\), \((s_B + 1)/2\) and \((s_A - s_B)/2\).

Note that in our model the price-sensitivity of liquidity-trader orders receives more stress than it does in much of the market-microstructure literature. The dealers in our formulation set their spread in order to exploit their monopoly power over part of the uninformed order flow. In stressing this aspect, our study resembles the important early paper by Copeland and Galai (1983).

In period 1, dealers may trade among themselves in such a way as to convey information
about the customer order received in 0. In period 2, each dealer again receives a customer order with probability 1/n which once more originates with an informed or an uninformed trader with probabilities $\alpha$ and $1 - \alpha$. Finally, we adopt the definition:

**Definition 1** A centralized market is one in which information on dealers’ order flow is freely and instantly available to other dealers. A decentralized market is one in which this is not the case.

The above structure of orders implies that our model is applicable to an extremely short period of time. To be specific, we analyze interdealer transactions that can occur in the moments between two substantial and possibly informative customer trades. Perhaps the best way to think of the model is as a description of the situation faced by dealers immediately following some event affecting the FX market.

### 2.2 The Static Problem

Before considering the more complicated dynamic problem of the dealers described above, let us study the static problem of a dealer who has a single opportunity to trade with informed and uninformed customers. Again assume that informed traders arrive with probability $\alpha$ and suppose that $q$ is the dealer’s current conditional probability for the event $\{z = 1\}$.

The intuition for some of the more important points that emerge from our analysis may be understood even within this simple framework. It is also important to understand the structure of the static problem since this is what dealers face in period 2 even when they have a fully dynamic problem to solve in period 0.

The dealer’s static value function $\Pi(s_A, s_B)$ may be written as:

$$
\Pi(s_A, s_B) \equiv \Pi_A(s_A) + \Pi_B(s_B) 
$$

where:

$$
\Pi_A(s_A) \equiv (1 - \alpha) \frac{(1 - s_A)}{2} (s_A - E(z)) + \alpha E \{ (s_A - z)1\{z = 1\} \} 
$$

$$
= (1 - \alpha) \frac{(1 - s_A)}{2} (s_A - (2q - 1)) + \alpha q (s_A - 1) 
$$
Π_B(s_B) \equiv (1 - \alpha)\left(\frac{1 + s_B}{2}\right) \left(E(z) - s_B\right) + \alpha E \left\{ (z - s_B)1\{z = -1\} \right\} \\
= (1 - \alpha)\left(\frac{1 + s_B}{2}\right) \left((2q - 1) - s_B\right) - \alpha(1 - q)(1 + s_B) \tag{3}

where we use the fact that E(z|q) = 2q - 1. The static value function is quadratic in the quotes, \( s_A \) and \( s_B \). Maximizing this function with respect to \( s_A \) and \( s_B \) yields the maximizing arguments:

\[
\begin{align*}
\hat{s}_A^* &= \min \left\{ \frac{q}{1 - \alpha}, 1 \right\} \\
\hat{s}_B^* &= \max \left\{ -\frac{1 - q}{1 - \alpha}, -1 \right\}
\end{align*}
\tag{4}
\]

The min and max operators in equation (4) appear because the maximum of the unconstrained value function may lie outside the interval \([-1, 1]\). In this case, the situation is as depicted in Figure 2, (b) and (c). This feature of the model will generate subcases for the various propositions we develop below.

In the absence of asymmetric information, i.e., when \( \alpha = 0 \), the optimal static quotes, \( s_A^* \) and \( s_B^* \), equal 1/2. In this case, the dealer’s calculation is solely motivated by the desire to exploit optimally the downward-sloping demand curve for liquidity-trader orders that he faces. For \( \alpha > 0 \), the absolute magnitude of the optimal quotes for a static dealer increases as he is now obliged to protect himself against informed trades by widening his spread.

Substituting for \( s_A^* = s_A(q) \) and \( s_B^* = s_B(q) \) in (2) and (3), we obtain that, for \( \alpha \leq 1/2 \):

\[
\Pi(s_A(q), s_B(q)) = \begin{cases} 
(1 - \alpha - q)^2/(2 - 2\alpha) & \text{if } 0 \leq q < \alpha \\
(2q(q - 1) + \alpha^2 + (1 - \alpha)^2)/(2 - 2\alpha) & \text{if } \alpha \leq q \leq (1 - \alpha) \\
(q - \alpha)^2/(2 - 2\alpha) & \text{if } (1 - \alpha) < q \leq 1
\end{cases}
\tag{5}
\]

One may note that \( \Pi \) is quadratic and continuous in \( q \), and that it has an interior minimum at \( q = 1/2 \). The form of \( \Pi \), which will be important for our results below, is shown in Figure 3, (a).

2.3 Filtering

Suppose now that the dealer trades in more than one period. In this case, he may be able to use the information he has gained from period 0 trades. The information is potentially valuable, first, because in his own period 2 trading, it may permit him to quote a bid-ask spread that yields higher expected profits. Second, he may be able to “sell” the information to other dealers in that they may be willing to trade with him at advantageous terms in the interbank market in order to learn about his order flow.
Suppose that dealers use Bayes’ rule to update their probability assessments. In this case, the conditional probability of the event \( \{ z = 1 \} \) following a buy order at 0, \( q_b \), will be:

\[
q_b \equiv \text{Prob}[ z = 1 \mid \text{buy}] = \frac{\alpha q + (1 - \alpha)q(1 - s_{A0})/2}{\alpha q + (1 - \alpha)(1 - s_{A0})/2}
\]

(6)

The updated probability for \( \{ z = 1 \} \) following a sell, \( q_s \), may be similarly derived as:

\[
q_s \equiv \text{Prob}[ z = 1 \mid \text{sell}] = \frac{(1 - \alpha)q(1 + s_{B0})/2}{\alpha(1 - q) + (1 - \alpha)(1 + s_{B0})/2}
\]

(7)

The important point to note about the updated probabilities is that they depend on the bid and ask quotes in the first period, \( s_{B0} \) and \( s_{A0} \). This dependency means that the dealer’s choice of period 0 quotes will be influenced by the impact of order flow information on period 2 profits. Since order flow information may be valuable to other dealers, the possibility of “selling” this information through trading in the interbank market will also affect the dealer’s optimal choice of period 0 quotes.

In standard, dynamic microstructure models such as those of Glosten and Milgrom (1985) or Easley and O’Hara (1987), this particular link between trading in different periods is broken by the fact that order flow information is assumed to be public knowledge.

### 2.4 Information Rents

Consider a dealer who has received no orders in period 0. If he trades again with outside customers in period 2 without receiving any information about other dealers’ trades, his expected profits will be \( \Pi(q) \). On the other hand, if he can buy information from another dealer who has received, for example, a buy order, his expected profits will be: \( \Pi(q_b) \).

The total increase in his expected profits when he learns of a buy order, \( \Pi(q_b) - \Pi(q) \), may be decomposed into a news effect and a “feedback rent”. Let \( \Pi(s_A(q), s_B(q)|q_b) \) be the expected profits the dealer obtains under updated probabilities, \( q_b \), but under the assumption that he sets \( s_A \) and \( s_B \) as if the probability of \( \{ z = 1 \} \) was \( q \). Now, one may write:

\[
\Pi(q_b) - \Pi(q) \equiv (\Pi(q_b) - \Pi(s_A(q), s_B(q)|q_b)) + (\Pi(s_A(q), s_B(q)|q_b) - \Pi(q))
\]

(8)

The first bracketed term on the right hand side is the expected value to the dealer of being able to adjust his quotes in response to the information. We refer to this extra value as the feedback rent.
This represents the increase in expected profits that the dealer can achieve by using the information to select his period 2 bid and ask quotes more efficiently. The second bracketed term in (8) is the pure news value of the information, i.e., the change in the dealer’s expected profits in a case in which he were, for some reason, unable to adjust his period 2 quotes. A similar decomposition can, of course, be performed for the change in expected profits due to information on a sale.

As mentioned above, our formulation of the interdealer market will entail informed dealers passing information to each other through their period 1 trades. The surplus over which they may be expected to bargain will then be the feedback rent, \( \Pi(q_b) - \Pi(s_A(q), s_B(q)|q_b) \). Note that even if \( \Pi(q_b) - \Pi(q) \) is negative, dealers will be willing to pay each other for the information so long as the feedback rent is positive. This is analogous to the willingness of someone to pay for news that he is going to die so as long as the knowledge will allow him to take actions that prolong his life at least a little. Now, in our case, the feedback rent, \( \Pi(q_b) - \Pi(s_A(q), s_B(q)|q_b) \) is non-negative as one may see from the fact that \( \Pi(q_b) = \max_{q \in [0,1]} \{ \Pi(s_A(q), s_B(q)|q_b) \} \). Hence, dealers will always be willing to pay for information.

We shall suppose that:

**Assumption 1.** Feedback rent associated with information on a customer trade is captured by the dealer who performs the trade.

This assumption has the merit of substantially simplifying the analysis. Though it represents a polar case, we think it unlikely that our results would be substantially affected if a more even division of feedback rents were allowed for.

### 2.5 The Dynamic Model

Suppose that a dealer who receives an order in period 0 is able to transact with a fraction, \( k \), of other market-makers in period 1. We think it reasonable to assume that \( k \) is closer to unity than zero. A substantial market-making operation with a sufficiently large dealing personnel can arrange simultaneous trades with fifteen to twenty other dealers. The number of international banks that trade in substantial size does not greatly exceed this figure.
Let \( P(A) \equiv [\alpha q + (1 - \alpha)(1 - s_{A0})/2] \) and \( P(B) \equiv [\alpha(1 - q) + (1 - \alpha)(1 + s_{B0})/2] \) denote the probabilities respectively of receiving an order at the ask or the bid in period 0. The period 0 value function for the dynamically optimizing dealer is:

\[
V_0(s_{A0}, s_{B0}) = \frac{1}{n} \left\{ \Pi_0(s_{A0}) + \Pi_0(s_{B0}) + P(A) \left( \frac{1}{n} \Pi_2(s_{A0}) + \frac{k(n - 1)}{n} rent(s_{A0}) \right) \right. \\
\left. + P(B) \left( \frac{1}{n} \Pi_2(s_{B0}) + \frac{k(n - 1)}{n} rent(s_{B0}) \right) \right. \\
\left. + (1 - P(A) - P(B)) \frac{1}{n} \Pi_2(s_{A2}, s_{B2}) \right\}
\]

where \( s_{A2} \) and \( s_{B2} \) are the optimal static, uninformed ask and bid quotes given in equation (4).

When dealers can trade with all other dealers in period 1, the dynamic value function simplifies to:

\[
V_0(s_{A0}, s_{B0}) = \frac{1}{n} \left\{ \Pi_0(s_{A0}) + \Pi_0(s_{B0}) + P(A) \Pi_2(s_{A0}) + P(B) \Pi_2(s_{B0}) + \\
(1 - P(A) - P(B)) \Pi_2(s_{A2}, s_{B2}) \right\}
\]

Since all dealers are the same, by the symmetry of the problem, each dealer’s value function equals \( 1/n \) times the expected value of the total market order flow. When \( k = 1 \), this is the above, simple, bracketed expression.

It is possible to obtain analytical solutions for the ask and bid quotes that maximize this value function since the first order conditions turn out to be cubic functions of the prices\(^4\). In fact, the complexity of the resulting expressions means that they are not of much practical use. However, as we show in the results section below, one can learn a considerable amount by analyzing the first order conditions and examining properties of the period 2 problem than by solving directly for bid and ask prices.

### 3 RESULTS

#### 3.1 Information and Expected Profits

In this section, we establish two propositions on the value of information in our model. We start with:
Proposition 1 The following three statements are equivalent.

1. News of a buy order increases total expected profits in period 2.

2. News of a buy order decreases dealers’ estimates of the conditional variance of the exchange rate.

3. \[ q \geq 1/2 \quad \text{or} \quad \alpha > \frac{(1/2 - q)(1 - q)}{1/2 - q + q^2} \] (11)

A similar result holds for news of a sell order.

To understand what drives this result, one may examine Figure 3. Panel (b) of the figure shows the total increase in expected profits associated with a buy order, \( \Pi(q_b) - \Pi(q) \). One may easily demonstrate that: \( q_b \geq q \) and \( q_s \leq q \). In the case depicted, the initial unconditional probability \( q \) equals \( q_0 \), while \( q_b > q_0 > 1/2 \). The fact that \( \Pi(q) \) is quadratic and has a minimum at 1/2 immediately implies that \( \Pi(q_b) - \Pi(q) > 0 \), i.e., information of a buy order implies higher expected profits. On the other hand, since in the diagram \( \Pi(q_s) < \Pi(q) \), it follows that news of a sale lowers expected profits.

Why does more information in the latter case lead to lower expected profits? The reason is as follows. One may show that the variance of the binomially distributed random variable, \( z \), equals \( 4q(1 - q) \) and that this has a maximum at \( q = 1/2 \). Thus, any information which implies a filtered, up-dated probability, \( q_u \), that lies closer to 1/2 than the original unconditional \( q \), also implies a increase in variance5.

But, higher variance lowers the dealer’s expected profits for the reason that the profit function (see equations (2) and (3)) is made up of kinked functions of the underlying payoff, \( z \). In this respect, the dealer’s profits resemble a short position in call and put options. Such claims are concave in the random payoff so, by Jensen’s inequality, adding uncertainty lowers expected value.

The pure news value of information in our model takes the simple form:

\[
\text{Pure News Value} \equiv (\Pi(q_b) - \Pi(q)) - (\Pi(q_b) - \Pi(s_A(q), s_B(q)|q_b)) = \frac{2q - 1}{1 - \alpha} (q_b - q) \quad (12)
\]

This immediately implies the following result:
Proposition 2 When buy and sell orders are equally probable, the pure news value of information is zero and the feedback rent associated with information equals the change in expected profits.

In other words, when \( q = 1/2 \), the pure news value is zero. To see the intuition behind this finding, suppose that the dealer cannot adjust his quotes in response to the information. Recall that the only three possible events are a buy order, a sell order or no trade. If there is no trade, the dealer still has \( \Pi(q) \). When \( q = 1/2 \), the problem is completely symmetric and, hence, information of either a bid or an ask order must change expected profits by the same amount. But, if all news has the same impact on expected value, it must be that the impact is zero. Hence, the only possible increase in value from the news must come from the dealer’s ability to adjust his quotes conditional on information. I.e., the increase in total value equals the feedback rent.

3.2 Bid-Ask Spreads

In this section, we show that the ask of a dynamically optimizing dealer is greater than that of a static dealer. Since, when \( n \) is large, dealers in a centralized market behave as though they are static, profit maximizers, this statement may be regarded as a statement about the behavior of dealer prices in centralized versus decentralized markets. We state the result formally as follows:

Proposition 3 The optimal period 0 ask of a dynamically optimizing dealer in the decentralized market exceeds that of the static solution. A corresponding result holds for decentralized market bids which exceed static bids in absolute magnitude.

By increasing the bid-ask spread from its static level, market-makers sacrifice short-term expected profits. On the other hand, however, they improve the quality of the information they derive from period 0 trades since uninformed trades are discouraged from transacting. Using the improved information, dealers can earn higher expected profits in subsequent trading.

Corollary 1 As \( n \to \infty \), the bid-ask spread is wider in a decentralized than in a centralized market.
3.3 Efficiency

In this section, we compare the efficiency of centralized and decentralized markets. It should be apparent that the two market organizations each have advantages and disadvantages. In a centralized market, dealers observe all period 0 customer trades so they can always update their subjective probability assessments and correspondingly adjust their period 2 bid and ask.

On the other hand, in a centralized market, the incentives of dealers to adjust their period 0 bid and ask so as to elicit an efficient amount of information are diluted. In the limit, as the number of dealers \( n \rightarrow \infty \), individual dealers in a centralized market will set \( s_{A0} \) and \( s_{B0} \) in a way that total ignores the informational rents associated with period 0 trading. In other words, they will act as though they are static profit maximizers.

In a decentralized market, the opportunity to sell information improves incentives to elicit information by optimal adjustment of the bid-ask spread. In the limit, when \( k = 1 \) and dealers can transact with all other market-makers, the entire market feedback rent associated with a period 0 customer order is captured by the dealer who receives the order. In this case, market-makers will optimally adjust their spreads to elicit information and hence the decentralized market will be privately efficient from the dealers’ collective point of view.

However, if dealers can only transact with a fraction of other market-makers, the advantage of better incentives will be reduced as it will then not be possible to capture the feedback rent associated with information. In addition, the decentralized market will suffer from the fact that dealers who do not receive inter-dealer trades will be unable to update their probabilities in response to period 0 order-flow.

To analyze this trade-off formally, we start with the lemma:

**Lemma 1** The unconditional expectation of period 2 profits is greater when dealers are able to update probabilities based on observation of period 0 order flow.

Lemma 1 demonstrates that the ability to adjust quotes in response to information on customer trades in period 0 increases the unconditional expectation of period 2 profits. Using this lemma, one may prove the following proposition.
Proposition 4 If dealers are able to transact with all other market-makers in the interval between customer trades, i.e., if \( k = 1 \), a decentralized market is fully efficient. If \( k \) is small, however, total expected dealer profits are higher in a centralized than a decentralized market.

In the remainder of the paper, we shall assume, for simplicity that \( k = 1 \).

3.4 Market Crashes

A point stressed by Glosten and Milgrom in their classic paper on dealer behavior was that markets with too many informed traders may collapse as dealers will be unable to make positive profits given the adverse selection problems they face. Such market crashes will involve collapses in volume as bid-ask spreads increase until the market closes. Note that it is possible for the market to close on one or both sides of the bid-ask spread. Crashes are costly because they undermine the informativeness of prices.

In our discussion of the static model in section 2, we have already implicitly considered such market collapses by discussing cases in which optimal ask and bid prices equal respectively plus and minus unity. Let us suppose, as seems reasonable, that:

Assumption 2 If \( s_A = 1 \) informed traders never buy while if \( s_B = -1 \) informed traders never sell.

Of course, in these cases, informed customers will be indifferent between trading and not trading, but any slight friction would be enough to make them strongly prefer not to trade.

The expressions for ask and bid prices in the static model, \( s_A = \min\{q/(1 - \alpha), 1\} \) and \( s_B = \max\{-(1 - q)/(1 - \alpha), -1\} \), immediately suggest under what conditions markets with static dealers will collapse. For any given \( q \), if \( \alpha \) is large enough, \( s_A = 1 \) and \( s_B = -1 \). On the other hand, for any given \( \alpha \), if \( q \to 1 \), eventually \( s_A = 1 \), while if \( q \to 0 \), eventually \( s_B = 1 \).

Notice from the discussion in the last paragraph that there are two reasons why the static market may collapse, either (i) too many informed traders (\( \alpha \) large for \( q \) around 1/2), or (ii) too little uncertainty about the value of the exchange rate (\( q \) close to zero or unity). (i) and (ii) lead
to qualitatively different outcomes in that in (i) both sides of the market closes while in (ii) only one side of the market crashes.

Panel (a) of Figure 4 illustrates the way in which in the static model, for a given $\alpha$, different assumptions on $q$ may generate crashes. In the case illustrated (in which $\alpha > 1/2$), for $\alpha < q < (1 - \alpha)$, the market crashes on both sides and expected profits are zero. For $q > \alpha$, the market crashes only on the ask side while for $q < (1 - \alpha)$, it crashes only on the bid.

One of the most interesting implications of our model is:

**Proposition 5** *The decentralized market never collapses in period 0.*

The interest of this result is that it suggests that decentralized markets are significantly more robust to the asymmetric information problems that provoke collapses in the static model. Recall that, as $n \to \infty$, dealers in centralized markets behave like static profit maximizers so once again this sheds light on the differences between centralized and decentralized market arrangements.

The intuition behind Proposition 5 is that in a decentralized market dealers have an incentive to provide a small but sufficient incentive for informed traders to transact and hence reveal their information. Given our assumptions about the price elasticity of orders by uninformed traders, if $s_{A0} = 1 - \epsilon$ or $s_{B0} = -1 + \epsilon$ for small, positive $\epsilon$, dealers can obtain very good information in the event of a buy or sell order respectively. As long as there is some rent to be extracted from this information in the form of higher period 2 profits, they will always have an incentive to open the market by adjusting their quotes enough to elicit trades from informed customers.

The above proposition is illustrated by panel (b) of Figure 4 which shows expected ask-side profits in the static and dynamic models as a function of the period 0 quote. The static expected profits equal zero at $s_{A0} = 1$ and unity is clearly the maximizing argument. The dynamic expected profits, which appear as a dotted line in the figure, are positive and increasing for ask quotes in an open interval below 1 and then, in fact, drop to zero at 1. The fact that they are positive for $s_{A0} = 1 - \epsilon$ for small, positive $\epsilon$ is what gives the above result.
3.5 Martingale Properties and Volatility

A feature of standard market microstructure models with competitive market-makers (see, for example, Glosten and Milgrom (1985) and Easley and O’Hara (1987)) is that bid and ask prices are martingales with respect to the information available to dealers. In this section, we shall see that in our decentralized market model, this is no longer the case and, in fact, bid and ask prices exhibit mean reversion as information is revealed.

Assumption 3 Suppose that interior solutions exist for the static model in period 0, i.e., that $\alpha < q < 1 - \alpha$.

First, consider the unconditional expectation of the difference between ask prices in periods 0 and 2.

Proposition 6 In the static model:

$$\mathbb{E}[s_{A2} - s_{A0}|S] = 0 \quad \mathbb{E}[s_{B2} - s_{B0}|S] = 0$$

while in the dynamic model:

$$\mathbb{E}[s_{A2} - s_{A0}|D] < 0 \quad \mathbb{E}[s_{B2} - s_{B0}|D] > 0$$

This proposition shows that weak-form market efficiency does not hold in our dynamic decentralized market while, in a centralized market with a large number of dealers, bids and asks will be martingales with respect to market-makers’ information. The basic feature of the model that permits deviations from martingale behavior is the market-power we assume for dealers. O’Hara (1994) comments on the fact that monopolistic elements can give such deviations.

Decentralized and centralized markets also differ in the amount of volatility they imply. One may show that the unconditional variance of changes in bid and ask is greater in the decentralized case.

Proposition 7 The unconditional variance of quote changes is greater in the dynamic than in the static case, i.e.,

$$\text{Var}(s_{A2} - s_{A0}|S) < \text{Var}(s_{A2} - s_{A0}|D)$$
$$\text{Var}(s_{B2} - s_{B0}|S) < \text{Var}(s_{B2} - s_{B0}|D)$$
3.6 Comparative Statics

Proposition 3 simply states that the bid-ask spread is larger in a decentralized than in a centralized market, without solving for the values taken by the bid and ask quotes. Though exact solutions for these quotes can be found, they are so complex that little more can be deduced. One may still, however, analyze the first order condition of the maximization problem to learn more about these solutions.

**Proposition 8** Let \( s_{A0}^* \) be an internal optimum for the ask price. Then, the following results hold:

\[
\frac{\partial s_{A0}^*}{\partial q} > 0 \quad \frac{\partial s_{A0}^*}{\partial \alpha} > 0 \quad \text{for } \alpha < q
\] (17)

As a first result, Proposition 8 shows that the ask price always increases when the probability of the event \( \{z = 1\} \) rises. Furthermore, an increase in the proportion of informed traders produces a rise in the ask. The reasons for this results are two: first, the need for protection against the informed traders is stronger; second, there is an incentive to increase the size of the bid-ask spread because with more informed traders it is possible to get more information.

**Proposition 9** Suppose that \( \alpha < q < 1 - \alpha < q_b \) and that the quotes chosen by the dealer are internal optima. Then the following results hold:

\[
\frac{\partial rent}{\partial q} < 0
\] (18)

\[
\frac{\partial rent}{\partial \alpha} > 0
\] (19)

The result on \( \partial rent/\partial q \) is interesting as it indicates that, for given \( \alpha \), when there is less uncertainty (\( q \) further from 1/2), there is less ex-ante information in the order flow, i.e., less opportunity to sell information.

3.7 Inter-Dealer Market Volume

When a market-maker has received some information, we assume that he can sell it in the second period to all the remaining market-makers. He accomplishes this through transacting on favorable
terms through the inter-dealer market. We suppose that market-makers will quote other dealers a bid-ask spread that: (i) is “regret free” according to the definition of Glosten and Milgrom, in that after a transaction market-makers do not regret having completed it; (ii) transfers feedback rent to the informed market-maker; (iii) creates no incentives for uninformed market-maker to pretend to possess information; and (iv) minimizes the quantity transacted.

The last property requires some comment. A given rent can be transferred between dealers by various combinations of price and quantity. In this sense, the prices quoted between dealers are indeterminate. However, (iv) implies unique inter-dealer quotes since if the transaction size is reduced too far and the spread made too generous, eventually uninformed dealers will be able to make profits masquerading as informed. Assuming that size of trades are reduced to a minimum implies that the incentive constraint implicit in (iii) must hold as an equality and hence determines inter-dealer quotes.

Now, suppose that market-maker 1 has received a buy order at time 0 (the same reasoning applies for a sell order) and wants to sell the information to market-maker 2. As he is willing to buy the currency and his expected value of the currency is $2q_b - 1$, market-maker 2 will quote an ask price in the interval $(2q - 1, 2q_b - 1)$ and a transaction will be completed for a quantity $\Delta x$ that conveys the rent of a buy order to market-maker 1 (subject to the assumed properties of inter-dealer trade ((i)-(iv)). Proposition 10 reports an interesting result concerning this value.

**Proposition 10** Let us define $\Delta x^*$ the minimum quantity transacted among two market makers. Suppose that the regularity conditions of Proposition 9 are satisfied. The following result holds:

$$\frac{\partial \Delta x^*}{\partial \alpha} > 0$$

(20)

The Proposition simply indicates that, given these conditions, the volume of transactions between market makers is indicative of the informativeness of the order flow as the minimum quantity transacted among two market makers is an increasing function of $\alpha$.

4 CONCLUSION

This paper has provided a theoretical analysis of a decentralized dealer market. Although our results are relevant to a broad category of markets in which order flow information is not publicly
available, the primary motivation for our study was the desire to understand price formation and efficiency in the foreign exchange market.

Our main findings are:

1. Bid ask spreads are wider in the decentralized market. The intuition here is that, by posting wider spreads, dealers can discourage price-sensitive liquidity traders and hence improve the informativeness of their order flow. The information embodied in orders can in turn be used to earn higher future profits and can be “sold” to other market-makers through interbank transactions at advantageous prices.

2. Decentralized markets are privately efficient from the collective point of view of market-makers when it is possible for dealers to transact with all other dealers in between potentially informative customer trades. This point underlines the potential importance of brokers as a way of facilitating large numbers of simultaneous transactions with other market-makers.

3. Decentralized markets are much less subject to market crashes than centralized markets. Information on order flow may be used to update subjective estimates of the underlying value of exchange rates. Even in circumstances in which static or centralized markets would crash due to excessive numbers of informed traders, dealers will have an incentive to preserve some turnover in the decentralized market as they can employ the information in the order flow in subsequent trading. Our model only allows two periods of trading with customers but we would conjecture that our results on crashes would hold in a multiperiod model, in that dealers would always have an incentive to preserve at least some order flow to gain information.

4. The time series behavior of exchange rates in our model differs according to whether trading is organized on a centralized or decentralized basis. When dealers maximize profits in a static fashion (which they will do in a centralized market containing large numbers of market-makers), bid and ask quotes are martingales with respect to the information available to dealers. In the decentralized market, bid-ask spreads on average shrink as order flow reveals information.

It is very interesting to note that this implication of the model is consistent with the findings of Goodhart and Figliuoli (1991). Their study suggests that prior to jumps in exchange rates, there is an increase in the negative autocorrelation. If we regard jump times as moments at
which significant information becomes public knowledge (i.e., corresponding to our period 3), then our model would suggest that in the immediately preceding period, a small number of agents will know the information and dealers will be adjusting quotes so that the bid-ask spread is contracting on average.

5. Another implication of the model for the statistical properties of exchange rate is that changes in rates will be more variable in the decentralized than in the centralized market. It is perhaps not clear quite what is the quantitative significance of this difference in variance but given the widely acknowledged volatility of exchange rates it is at least reassuring that our model predicts greater variance in decentralized markets.
5 APPENDIX

Proofs are stated for the ask side of the market throughout. Similar arguments apply to the bid side.

5.1 Proof of Proposition 1

Proposition 1 The following three statements are equivalent.

1. News of a buy order increases total expected profits in period 2.
2. News of a buy order decreases dealers’ estimates of the conditional variance of the exchange rate.
3. 
   \[ q \geq \frac{1}{2} \quad \text{or} \quad \alpha > \frac{(1/2 - q)(1 - q)}{1/2 - q + q^2} \]  

A similar result holds for news of a sell order.

Proof. The equivalence of the first two statements is obvious; in fact: as \( \Pi(s_A(q), s_B(q)) \) is symmetric around 1/2 we have a gain in the expected profits from a buy order if \( q_b - 1/2 > 1/2 - q \); that corresponds to a reduction in the conditional variance of the exchange rate. Moreover, \( q_b - 1/2 > 1/2 - q \) holds if:

\[ \alpha q^2 + (1 - \alpha)(1 - s_{A0})(q - 1/2) > 0 \]  

Then, for \( s_{A0} = q/(1 - \alpha) \), this condition becomes:

\[ \alpha q^2 > (1 - \alpha - q)(1/2 - q) \]

It is immediately obvious that this condition holds for the values of \( \alpha \) and \( q \) respecting the condition (21). This completes the proof. □

5.2 Proof of Proposition 3

Proposition 3 The optimal period 0 ask of a dynamically optimizing dealer in the decentralized market exceeds that of the static solution. A corresponding result holds for decentralized market bids which exceed static bids in absolute magnitude.
Proof. Let $V'_0$ be that part of the dynamic value function which depends on the ask price, $s_{A0}$, multiplied by the constant $n$. $V'_0$ equals:

$$V'_0(s_{A0}) \equiv \Pi_0(s_{A0}) + \left(\frac{n - 1}{n}\right)\text{Prob}(A)\Pi_2(s_{A0}) - \frac{1}{n} \left[(n - 1)\text{Prob}(A) + \frac{1 - s_{A0}}{2}(1 - \alpha)\right] \Pi_2(s_{A2}, s_{B2})$$

(23)

Assuming that $\alpha < 1/2$ and given the different form of the static value function for different configurations of $q$ and $\alpha$, we consider six cases:

$q < q_b < \alpha$; $q < \alpha \leq q_b < 1 - \alpha$; $q < \alpha < 1 - \alpha \leq q_b$; $\alpha < q < q_b < 1 - \alpha$; $\alpha < q < 1 - \alpha \leq q_b$ and $q > 1 - \alpha$ so that $q_b > 1 - \alpha$. In all six cases, we have:

$$\frac{\partial V'}{\partial s_{A0}} = \frac{\partial \Pi_0(s_{A0})}{\partial s_{A0}} + \left(\frac{\partial}{\partial s_{A0}} \left[\text{Prob}(A)\Pi_2(s_{A0})\right]\right) + \frac{1 - \alpha}{2} \Pi_2(s_{A2}, s_{B2})$$

$$= q - (1 - \alpha)s_{A0} + \Gamma = 0$$

(24)

We show that in all cases $\Gamma$ is positive and therefore:

$$s_{A0}^* \geq \text{the static solution}$$

(25)

In particular, we show that in the first five cases:

$$s_{A0}^* > \frac{q}{1 - \alpha} = \text{the static solution}$$

(26)

while in the last case the static and the dynamic solutions are both equal to one.

**Case 1:** $q < q_b < \alpha$. The components of $\Gamma$ are as follows:

$$\frac{1 - \alpha}{2} \Pi_2(s_{A2}, s_{B2}) = \frac{[1 - \alpha - q]^2}{4}$$

(27)

$$\text{Prob}(A) \frac{\partial \Pi_2(s_{A0})}{\partial s_{A0}} = \frac{(q_b - q)(q_b - (1 - \alpha))}{2}$$

(28)

$$\frac{\partial \text{Prob}(A)}{\partial s_{A0}} \Pi_2(s_{A0}) = -\frac{[1 - \alpha - q_b]^2}{4}$$

(29)

Since:

$$\frac{\partial q_b}{\partial s_{A0}} = \frac{(q_b - q)(1 - \alpha)}{2\text{Prob}(A)}$$

(30)

$$\frac{\partial \text{Prob}(A)}{\partial s_{A0}} = -\frac{1 - \alpha}{2}$$

(31)

It then follows that:

$$\Gamma \equiv \frac{(q_b - q)^2}{4}$$

(32)

**Case 2:** $q < \alpha \leq q_b < 1 - \alpha$.

$$\frac{1 - \alpha}{2} \Pi_2(s_{A2}, s_{B2}) = \frac{[1 - \alpha - q]^2}{4}$$

(33)

$$\text{Prob}(A) \frac{\partial \Pi_2(s_{A0})}{\partial s_{A0}} = \frac{(2q_b - 1)(q_b - q)}{2}$$

(34)

$$\frac{\partial \text{Prob}(A)}{\partial s_{A0}} \Pi_2(s_{A0}) = \frac{1 - q_b}{2}q_b - \frac{\alpha^2 + (1 - \alpha)^2}{4}$$

(35)
We end up with:

$$\Gamma \equiv \frac{2(q_b - q)^2 - (q - \alpha)^2}{4}$$

This is positive as $q_b > q$ and $\alpha > q$.

**Case 3:** $q \alpha < 1 - \alpha < q_b$. Using the fact that:

$$\begin{align*}
\frac{1 - \alpha}{2} \Pi_2(s_{A2}, s_{B2}) &= \frac{[(1 - \alpha) - q]^2}{4} \\
\text{Prob}(A) \frac{\partial \Pi_2(s_{A0})}{\partial s_{A0}} &= \frac{(q_b - \alpha)(q_b - q)}{2} \\
\frac{\partial \text{Prob}(A)}{\partial s_{A0}} \Pi_2(s_{A0}) &= -\frac{(q_b - \alpha)^2}{4}
\end{align*}$$

It follows that:

$$\Gamma \equiv \frac{((q_b - \alpha)^2 + 2(q_b - \alpha)(\alpha - q) + [(1 - \alpha) - q]^2)}{4}$$

That is positive as $q_b > \alpha$ and $\alpha > q$.

**Case 4:** $\alpha < q < q_b < 1 - \alpha$. We now have:

$$\begin{align*}
\frac{1 - \alpha}{2} \Pi_2(s_{A2}, s_{B2}) &= \frac{\alpha^2 + (1 - \alpha)^2 - (1 - q)q}{4} \\
\text{Prob}(A) \frac{\partial \Pi_2(s_{A0})}{\partial s_{A0}} &= \frac{(2q_b - 1)(q_b - q)}{2} \\
\frac{\partial \text{Prob}(A)}{\partial s_{A0}} \Pi_2(s_{A0}) &= -\frac{(q_b - \alpha)^2}{4}
\end{align*}$$

Hence:

$$\Gamma \equiv \frac{(q_b - q)^2}{2}\frac{1}{4}$$

**Case 5:** $\alpha < q < 1 - \alpha \leq q_b$. We have that:

$$\begin{align*}
\frac{1 - \alpha}{2} \Pi_2(s_{A2}, s_{B2}) &= \frac{\alpha^2 + (1 - \alpha)^2 - (1 - q)q}{4} \\
\text{Prob}(A) \frac{\partial \Pi_2(s_{A0})}{\partial s_{A0}} &= \frac{(q_b - \alpha)(q_b - q)}{2} \\
\frac{\partial \text{Prob}(A)}{\partial s_{A0}} \Pi_2(s_{A0}) &= -\frac{(q_b - \alpha)^2}{4}
\end{align*}$$

this implies that:

$$\Gamma \equiv \frac{2q_b(q_b - q) + [(1 - \alpha) - q]^2}{4}$$

But, since $q_b > 1 - \alpha > q$, it follows that $q_b^2 > - (1 - \alpha)[2q - (1 - \alpha)]$. So $\Gamma > 0$ as required. Finally, consider the last case:

**Case 6:** $q > 1 - \alpha$ so that $q_b > 1 - \alpha$. Here we have:

$$\begin{align*}
\frac{1 - \alpha}{2} \Pi_2(s_{A2}, s_{B2}) &= \frac{(q - \alpha)^2}{4} \\
\text{Prob}(A) \frac{\partial \Pi_2(s_{A0})}{\partial s_{A0}} &= \frac{(q_b - \alpha)(q_b - q)}{2} \\
\frac{\partial \text{Prob}(A)}{\partial s_{A0}} \Pi_2(s_{A0}) &= -\frac{(q_b - \alpha)^2}{4}
\end{align*}$$

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Therefore:

\[ \Gamma \equiv \left[ \frac{2(q_b - \alpha)(q_b - q) + (q - \alpha)^2 - (q_b - \alpha)^2}{4} \right] \]

\[ = \left[ \frac{2q_b(q_b - q) + q^2 - q_b^2}{4} \right] = \frac{(q_b - q)^2}{4} > 0 \] (52)

This completes the proof. \( \square \)

### 5.3 Proof of Efficiency Results

**Lemma 1** The unconditional expectation of period 2 profits is greater when dealers are able to up-date probabilities based on observation of period 0 order flow.

**Proof.** Define \( E_{\Pi_u} \) as the unconditional expectation of period 2 profits when probabilities are updated at 1 after the observation of a trade should one occur.

\[
E_{\Pi_u} = \text{Prob(buy)} \left[ \max_{s_{A2}, s_{B2}} \Pi(s_{A2}, s_{B2} | q_b) \right] + \text{Prob(sell)} \left[ \max_{s_{A2}, s_{B2}} \Pi(s_{A2}, s_{B2} | q_s) \right] + [1 - \text{Prob(buy)} - \text{Prob(sell)}] \left[ \max_{s_{A2}, s_{B2}} \Pi(s_{A2}, s_{B2} | q) \right]
\] (53)

The unconditional expectation of profits without updating is denoted: \( E_{\Pi} \). By the iterative property of Conditional Expectations this can be expressed in the following way:

\[
E_{\Pi} = \max_{s_{A2}, s_{B2}} \left[ \text{Prob(buy)} \Pi(s_{A2}, s_{B2} | q_b) + \text{Prob(sell)} \Pi(s_{A2}, s_{B2} | q_s) + [1 - \text{Prob(buy)} - \text{Prob(sell)}] \Pi(s_{A2}, s_{B2} | q) \right]
\] (54)

It is then clear that \( E_{\Pi_u} > E_{\Pi} \) for \( \text{Prob(buy)} + \text{Prob(sell)} > 0 \). \( \square \)

**Proposition 4** If dealers are able to transact with all other market-makers in the interval between customer trades, then a decentralized market is fully efficient as all dealers up-date probability assessments optimally and bid and ask prices are adjusted so as to maximize the total expected profits of all dealers. Otherwise, the centralized market trading arrangements may imply higher total expected dealer profits than decentralized trading.

**Proof.** Follows from Lemma 1. \( \square \)

### 5.4 Proof of Proposition 5

**Proposition 5** The decentralized market never collapses in period 0.
Proof. We can prove this statement in two steps. In the first, we prove that the market cannot be always completely closed in period two and at least on one side in period 0. In the second we prove that if the market is open at least on one side in the second period it cannot be closed on either side in period 0.

STEP 1. Suppose that the market-makers strategy implies that the market is always closed in period 2 and is closed on the sell side in period 0; we show there exists another strategy that dominates it. Suppose we fix in period 0 $s_{A0} = 1$ and $s_{B0} = -1 + \epsilon$, for $\epsilon > 0$ and small. Suppose that a dealer will set $s_{A2} = 1/2$ and $s_{B2} = -1$ if he receives a sell order and $s_{A2} = 1$ and $s_{B2} = -1$ otherwise. As the sell order in period 0 for a given dealer will occur with probability $\alpha(1-q)/n$ and as the bid-ask spread will have a negligible impact on period 0 profits the total expected profits will be: $(1-\alpha)(1-s_{A2})(1+s_{A2})/2 = 3(1-\alpha)/8$; this implies that the second strategy dominates the first one. A similar argument works for the ask side.

STEP 2. Suppose the ask side is closed in 0. Lowering $s_{A0}$ slightly hardly affects period 0 expected profits but means that, in the event of a buy order, the dealer receiving it knows almost surely that $z = 1$. The profit function with the ask side closed in 0 is:

$$V_0(s_{A0}, s_{B0}) = V_0(1, s_{B0}) = \frac{1}{n} \left[ \Pi_0(s_{B0}) + P(A)\Pi_2(s_{B0}) + (1 - P(A))\Pi_2(s_{A2}, s_{B2}) \right]$$  \hspace{1cm} (55)

with $s_{A0} = 1 - \epsilon$ we have:

$$V_0(s_{A0}, s_{B0}) = V_0(1 - \epsilon, s_{B0}) = \frac{1}{n} \left[ \Pi_0(s_{B0}) + P(A)\Pi_2(s_{A0}|s_{A0}=1-\epsilon) + P(B)\Pi_2(s_{B0}) + (1 - P(A) - P(B))\Pi_2(s_{A2}, s_{B2}) \right]$$  \hspace{1cm} (56)

Hence, the result follows if $\Pi_2(s_{A0}|s_{A0}=1-\epsilon) > \Pi_2(s_{A2}, s_{B2})$. But, as long as $q$ is different from 1,

$$\Pi_2(s_{A0}|s_{A0}=1-\epsilon) = \Pi_2(\hat{q}|\hat{q}=1) > \Pi_2(q) = \Pi_2(s_{A2}, s_{B2})$$  \hspace{1cm} (57)

Therefore, the ask side of the market will not be always closed in period 2; a similar argument holds for the bid side. This completes the proof. □

5.5 Statistics of Quote Changes

Proposition 6 In the static model:

$$\mathbb{E}[s_{A2} - s_{A0}|S] = 0 \hspace{1cm} \mathbb{E}[s_{B2} - s_{B0}|S] = 0$$  \hspace{1cm} (58)

while in the dynamic model:

$$\mathbb{E}[s_{A2} - s_{A0}|D] < 0 \hspace{1cm} \mathbb{E}[s_{B2} - s_{B0}|D] > 0$$  \hspace{1cm} (59)
Proof. In the dynamic model, we can write the difference in expectations as:

\[
E[s_{A2} - s_{A0}|D] = \text{Prob}(\text{sell}) \left( \frac{q_s}{1 - \alpha} - s_{A0} \right) + \text{Prob}(\text{buy}) \left( \frac{q_b}{1 - \alpha} - s_{A0} \right) \\
+ \text{Prob}(\text{no trade}) \left( \frac{q}{1 - \alpha} - s_{A0} \right)
\]

\[
= \frac{q}{1 - \alpha} - s_{A0} < 0
\] (60)

where we use the fact that \(\text{Prob}(\text{sell})(qs - q) = -\text{Prob}(\text{buy})(qb - q)\) and \(s_{A0} > q/(1 - \alpha)\). □

Proposition 7 The unconditional variance of quote changes is greater in the dynamic than in the static case, i.e.,

\[
\text{Var}(s_{A2} - s_{A0}|S) < \text{Var}(s_{A2} - s_{A0}|D) \\
\text{Var}(s_{B2} - s_{B0}|S) < \text{Var}(s_{B2} - s_{B0}|D)
\]

Proof. Consider volatility with the two sets of quote setting behavior. In both static and dynamic cases,

\[
\text{Var}(s_{A2} - s_{A0}) = \text{Prob}(\text{sell}) \left[ \frac{q_s - q}{1 - \alpha} \right]^2 + \text{Prob}(\text{buy}) \left[ \frac{q_b - q}{1 - \alpha} \right]^2 \\
= \left[ \frac{\alpha q(1 - q)}{1 - \alpha} \right]^2 \left( \frac{1}{\text{Prob}(\text{buy})} + \frac{1}{\text{Prob}(\text{sell})} \right)
\] (61)

The result then follows from the fact that \(\text{Prob}(\text{buy})\) and \(\text{Prob}(\text{sell})\) are larger in the static than the dynamic case. □

5.6 Proof of Comparative Statics

Proposition 8 Let \(s_{A0}^*\) be an internal optimum for the ask price. Then, the following results hold:

\[
\frac{\partial s_{A0}^*}{\partial q} > 0 \quad \frac{\partial s_{A0}^*}{\partial \alpha} > 0 \quad \text{for } \alpha < q
\] (62)

Proof. We can use the first order condition to study the effect of a change in any parameter of the model, \(\beta\), on \(s_{A0}^*\), as the second order condition guarantees we still have an internal solution; therefore we consider:

\[
\frac{\partial s_{A0}^*}{\partial \beta} = -\frac{\partial^2 V'/\partial s_{A0}^*}{\partial^2 V'/\partial s_{A0}^*}
\] (63)

Now, as from the second order condition it follows that \(\partial^2 V'/\partial s_{A0}^* < 0\), the sign of the derivative of \(s_{A0}^*\) with respect to \(\beta\) corresponds to that of the numerator. Hence, consider \(\partial^2 V'/\partial q \partial s_{A0}^*\), that is equal to

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To prove this is always positive, we have to consider five of the six cases discussed in Proposition (2), because for \( q > 1 - \alpha \) we do not have an internal optimum. It is easy to show that

\[
\frac{\partial (q_b - q)}{\partial q} = \frac{\alpha (1 - q - q_b)}{\Delta}
\]

where \( \Delta = \alpha q + (1 - \alpha)(1 - s_{A0})/2 \); for \( q + q_b > 1 \), as \( 1 - q_b > 0 \) and \( \Delta > \alpha q \) this derivative is negative, but larger than \(-1\); while, for \( q + q_b < 1 \) this derivative is positive. This permits to show that in all cases \( \partial \Gamma / \partial q > -1 \).

In Case 1:

\[
\frac{\partial \Gamma}{\partial q} = \frac{(q_b - q) q_b - q}{2}
\]

Therefore \( \partial \Gamma / \partial q > 0 \) as \( q + q_b < 1 \).

In Case 2:

\[
\frac{\partial \Gamma}{\partial q} = \frac{(q_b - q) (q_b - q) + \alpha - q}{2}
\]

Therefore \( \partial \Gamma / \partial q > 0 \) as \( q + q_b < 1 \) and \( \alpha > q \).

In Case 3:

\[
\frac{\partial \Gamma}{\partial q} = \frac{(q_b - q) (q_b - q) - [(1 - \alpha) - q] / 2}{\Delta^2} > -1
\]

In Case 4:

\[
\frac{\partial \Gamma}{\partial q} = (q_b - q) \frac{\partial (q_b - q)}{\partial q} + q \frac{\partial q_b}{\partial q} - (1 - \alpha) / 2 > -1
\]

Finally in Case 5, we have:

\[
\frac{\partial \Gamma}{\partial q} = \{(q_b - q) \frac{\partial (q_b - q)}{\partial q} + q \frac{\partial q_b}{\partial q} - (1 - \alpha)\} / 2 > -1
\]

Let us consider \( \frac{\partial^2 V^*}{\partial q \partial \alpha} = s_{A0}^* + \partial \Gamma / \partial \alpha \) In this case, we have that:

\[
\frac{\partial q_b}{\partial \alpha} = \frac{(1 - s_{A0}^*) q (1 - q)}{2 \Delta^2} > 0
\]

For \( \alpha < q \) and \( q_b < 1 - \alpha \), we have to discuss only Case 4; we can easily prove that \( s_{A0}^* + \partial \Gamma / \partial \alpha \) is positive. In fact, we have:

\[
\frac{\partial \Gamma}{\partial \alpha} = (q_b - q) \frac{\partial q_b}{\partial \alpha} > 0
\]

\( \Box \)

**Proposition 9** Suppose that \( \alpha < q < 1 - \alpha < q_b \) and that the quotes chosen by the dealer are internal optima. Then the following results hold:

\[
\frac{\partial \text{rent}}{\partial q} < 0 \\
\frac{\partial \text{rent}}{\partial \alpha} > 0
\]
Proof. We assume that $\alpha$ is such that $\alpha < q < q_b < 1 - \alpha$ so that we concentrate on Case 4. The rent from a buy order is given by:
\[
\text{rent}(\text{buy}) = \frac{(q_b - q)^2}{1 - \alpha}
\] (72)
In Case 4, $q + q_b > 1$ so that $\partial(q_b - q)/\partial q$ is negative. This is sufficient to prove that $\partial\text{rent}(\text{buy})/\partial q$ is negative. Conversely, as $\partial q_b/\partial \alpha > 0$ it is immediately obvious that $\partial\text{rent}(\text{buy})/\partial \alpha$ is positive. □.

5.7 Proof of Gearing Effect Results

Proposition 10 Let us define $\Delta x^*$ the minimum quantity transacted among two market makers. Suppose that the regularity conditions of Proposition 9 are satisfied. The following result holds:
\[
\frac{\partial \Delta x^*}{\partial \alpha} > 0
\]
Proof. We assume that $\alpha$ is such that $\alpha < q < q_b < 1 - \alpha$ so that we concentrate on Case 4. If a buy order has been received, the minimum value of the transacted quantity is:
\[
\Delta x^* = \frac{\text{rent}(\text{buy})}{2(q_b - q)}
\] (73)
We know that the rent of a buy order is given in Case 4 by:
\[
\text{rent}(\text{buy}) = \frac{(q_b - q)^2}{1 - \alpha}
\] (74)
This implies:
\[
\Delta x^* = \frac{(q_b - q)}{2(1 - \alpha)}
\] (75)
Therefore, as $\partial q_b/\partial \alpha > 0$, it follows that $\partial\Delta x^*/\partial \alpha > 0$. This completes the proof. □

FOOTNOTES

1) Their model has a batch trading structure with a single price rather than a bid-ask spread and risk pooling plays an important role.
2) The latter is simply a normalization as we could value assets relative to the value of a safe bond.
3) As in any dynamic programming problem under uncertainty, the dealer will do better if he can employ “feedback” controls that adjust according to information received.
4) Though complicated, closed form solutions to cubic equations are available (see Abramovitz and Stegun (1964)).
5) Readers more familiar with normal filtering problems may find this slightly surprising as up-dating, in that case, reduces uncertainty. In the present context, for certain values of $q$, up-dating actually increases the conditional variance.
References


Figure 1: STOCHASTIC ORDER FLOW STRUCTURE

\[ t=0 \]

START

\[ \frac{n-1}{n} \]
\[ \frac{1}{n} \]

No Customer for Dealer \( i \)

Order Informed

\( \alpha \)

Customer for Dealer \( i \)

Order Uninformed

\( 1-\alpha \)

Buy

Sell

\( q \)

\( 1-q \)

\( \frac{1-s_A}{2} \)

\( \frac{1+s_B}{2} \)

\( \frac{s_A-s_B}{2} \)

No Trade

Update \( q \). If \( t < 2 \) interbank trade and go to START. Else stop.

If \( t < 2 \) go to START. Else stop.
Figure 4: MARKET CRASHES AND INFORMATION